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XXXIII Cycle

A Diffusion MRI Consistent Model for the Efficient Solution of the Anisotropic Forward Problem in EEG

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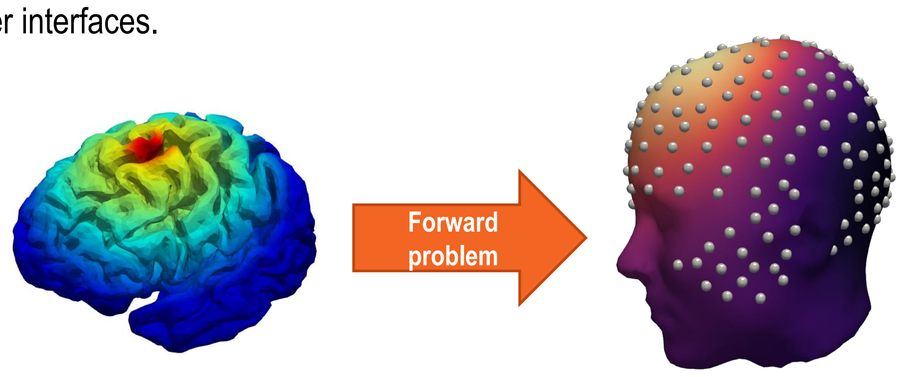
Research context and motivation

- The forward problem of **electroencephalography** (EEG) is the cornerstone for **brain analysis** in a wide range of medical and research applications, including epilepsy treatment, transcranial brain stimulation, electric impedance tomography, and brain computer interfaces.



- Electromagnetic solvers are used in practice to obtain the solution of the EEG forward problem. Although popular in the biomedical community, the integral equation-based **Boundary Element Method** (BEM) relies on a piecewise-uniform conductivity assumption, oversimplifiying the electrical properties of several head tissues and resulting in important modeling errors.
- We wish to develop a new method that can fully exploit advanced multimodal **MRI** and **tractography** technologies to obtain an efficient and realistic forward problem solution.

Addressed research questions/problems

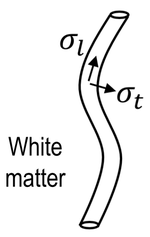
- The formulation of the EEG forward problem is given by the **anisotropic Poisson's equation** in the quasi-static regime, relating a known primary activity $\boldsymbol{J}_p$ to the unknown electrical potential ϕ

$$\nabla \cdot (\bar{\bar{\sigma}} \nabla \phi) = \nabla \cdot \boldsymbol{J}_p$$

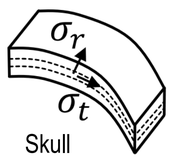
- The **boundary conditions** ensure the continuity of the potential and current density

$$\phi|_i^- = \phi|_i^+, \quad \boldsymbol{r} \in \Gamma_i$$
$$\hat{\boldsymbol{n}} \cdot \bar{\bar{\sigma}} \nabla \phi|_i^- = \hat{\boldsymbol{n}} \cdot \bar{\bar{\sigma}} \nabla \phi|_i^+, \quad \boldsymbol{r} \in \Gamma_{i<N}$$
$$\hat{\boldsymbol{n}} \cdot \bar{\bar{\sigma}} \nabla \phi = 0, \quad \boldsymbol{r} \in \Gamma_N$$

- The forward problem can be solved in terms of **surface integral equations** only on the condition that the head can be approximated as a piecewise-uniform medium. However, the **skull** and **white matter** in particular are strongly inhomogeneous and anisotropic.



Tissue	W. Matter	G. Matter	CSF	Skull	Scalp
Isotropy	✗	✓	✓	✗	✓
Conductivity (S/m)	$\sigma_l = 1.13$ $\sigma_t = 0.13$	$\sigma = 0.33$	$\sigma = 1.79$	$\sigma_r = 0.01$ $\sigma_t = 0.001$	$\sigma = 0.43$



- Until now, expensive full volumetric methods such as the Finite Element Method (FEM) are required to incorporate **anisotropic** and **inhomogeneous conductivities**. Furthermore, the diffusion tensor imaging-based anisotropy modeling of the white matter with classical volume elements remains challenging to validate.

Future work

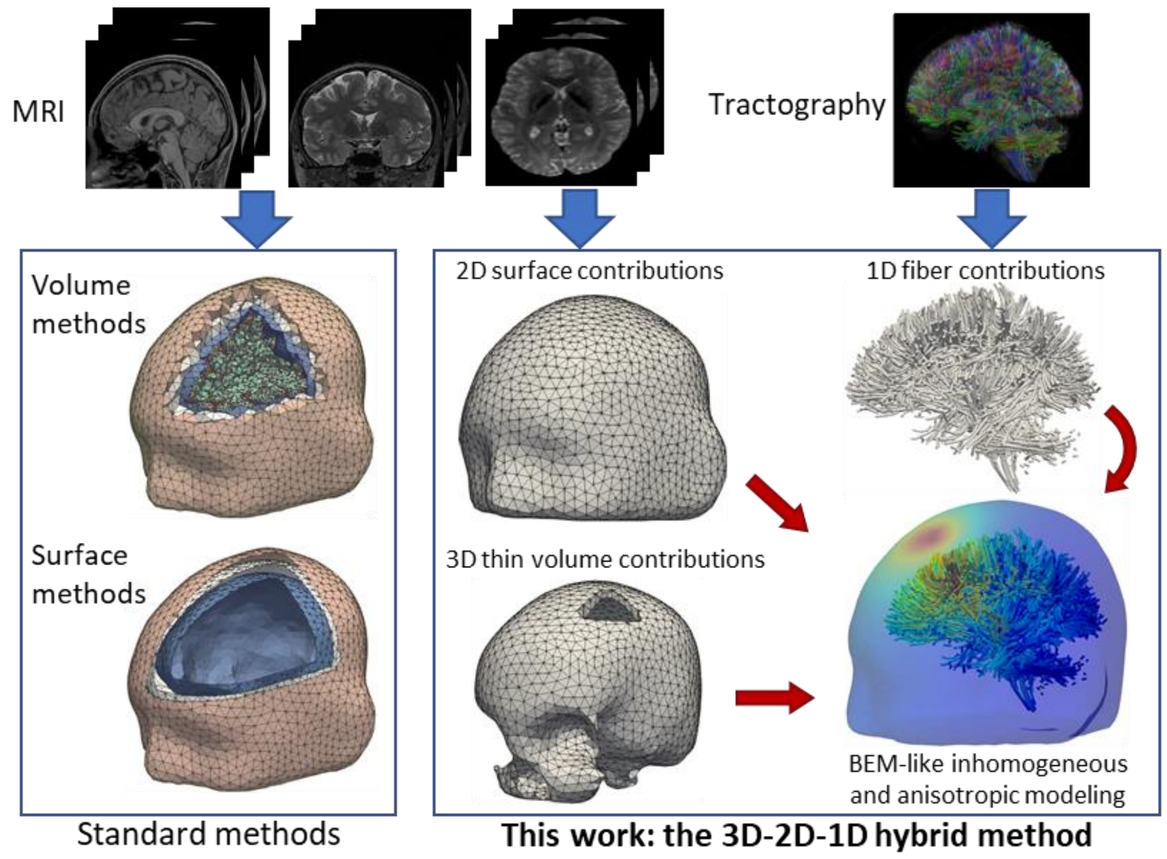
- Integration of a **fast method** to accelerate both matrix building and inversion and reach high computational efficiency

Submitted and published works

- M.Y. Monin, L. Rahmouni, and F.P. Andriulli, "*A Hybrid Integral Equation Approach to Solve the Anisotropic Forward Problem in Electroencephalography*", IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting, Boston, 2018, pp. 2395-2396
- M.Y. Monin, L.Rahmouni, and F.P. Andriulli "*Diffusion MRI Consistent Wire Models For Efficient Solutions of the Anisotropic Forward Problem in EEG"*, International Conference on Electromagnetics in Advanced Applications, Granada, 2019
- M.Y. Monin, L.Rahmouni, A. Merlini and F.P. Andriulli, "*A Hybrid Volume-Surface-Fiber Integral Equation for the Anisotropic Forward Problem in Electroencephalography*", IEEE Journal of Electromagnetics, RF and Microwaves in Medicine and Biology, Submitted

Adopted methodologies

- The anisotropic and inhomogeneous conductivity is explicitly modeled in terms of **equivalent currents**, leading to anatomically matching **volume contributions** in the skull and **fiber contributions** in the white matter.

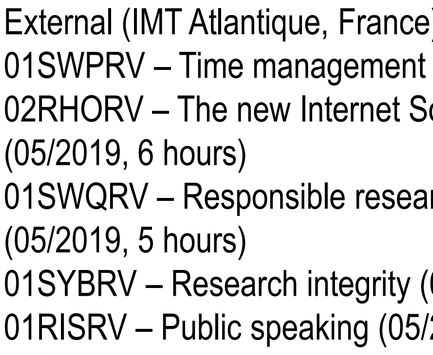


Novel contributions

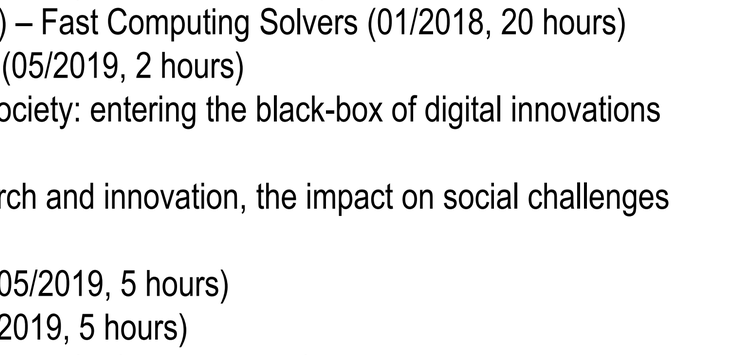
- The new **fiber** and **volume** contributions are coupled with the **surface** contributions of the classical BEM to obtain an efficiently discretized **hybrid system** of integral equations

$$\begin{matrix} \text{2D} \\ \text{1D} \\ \text{3D} \end{matrix} \begin{bmatrix} -\boldsymbol{G}_{pp} + \boldsymbol{D}^*_{pp} & -\boldsymbol{D}^*_{v\,ph} & \boldsymbol{G}_{ps} - \boldsymbol{D}^*_{v\,ps} \\ -\boldsymbol{S}_{hp} & \boldsymbol{G}_{v\,hh} + \boldsymbol{S}^*_{v\,hh} & \boldsymbol{S}^*_{v\,hs} \\ -\boldsymbol{S}_{sp} & \boldsymbol{S}^*_{v\,sh} & \boldsymbol{G}_{v\,ss} + \boldsymbol{S}^*_{v\,ss} \end{bmatrix} \begin{bmatrix} \alpha_p \\ \alpha_h \\ \alpha_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_p \\ \boldsymbol{v}_h \\ \boldsymbol{v}_s \end{bmatrix}$$

- Numerical results demonstrate both its **accuracy** and **applicability** in real-case scenario



3-layer canonical model



Comparison with anisotropic DTI FEM solver

List of attended classes

- 01MMRRV – Advanced Computational Electromagnetics for Antenna Analysis and Design (03/2019, 20 hours)
- 01QTEIU – Data mining concepts and algorithms (03/2018, 20 hours)
- 01SVFRV – Metamaterials: Theory and multiphysics applications (03/2018, 20 hours)
- External (European School of Antennas, Italy Advanced Computational Electromagnetics (09/2018, 37 hours)
- External (European School of Antennas, Italy) Microwave Imaging and Diagnostics: Theory, Techniques, and Applications (03/2018, 30 hours)
- External (IMT Atlantique, France) – Fast Computing Solvers (01/2018, 20 hours)
- 01SWPRV – Time management (05/2019, 2 hours)
- 02RHORV – The new Internet Society: entering the black-box of digital innovations (05/2019, 6 hours)
- 01SWQRV – Responsible research and innovation, the impact on social challenges (05/2019, 5 hours)
- 01SYBRV – Research integrity (05/2019, 5 hours)
- 01RISRV – Public speaking (05/2019, 5 hours)
- 01SHMRV – Entrepreneurial Finance (05/2019, 5 hours)
- 02LWHRV – Communication (05/2019, 5 hours)

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