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XXXIII Cycle

Quasi-Helmholtz Projector Based Full Wave Solver Encompassing Eddy Currents

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Research context and motivation

- **Context:** **Eddy current** problems appear in various industrial applications such as non destructive testing, induction heating, etc. From a modeling perspective, such scenarios involve **lossy conductors**, characterized by a **complex permittivity**.

  → Solvers **tailored to eddy currents** can be obtained from an approximation of Maxwell's equations.

- **Motivation:** Pure **eddy current models**, while effective, are unfortunately **limited** to low frequencies. On the other hand, the widespread **PMCHWT** formulation is **ill-conditioned** in eddy current conditions.

  → Simulations in **wide frequency** ranges therefore require the use of **separate solvers**, which is highly inconvenient.

- **Goal:** Propose a **unified**, **full wave** formulation to smoothly transition between high and low frequencies, **including eddy current** scenarios. The solver should be **well-conditioned**, thus producing **fast** and **accurate** solutions.

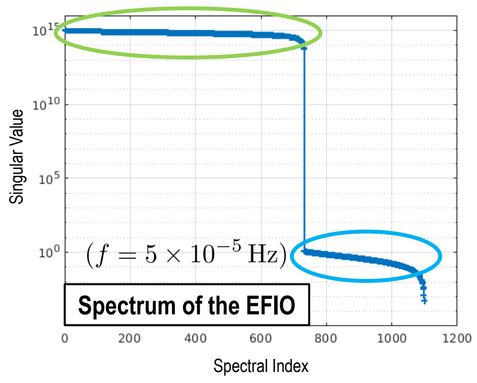
Addressed research questions/problems

- **PMCHWT Equation:** Classic discretization scheme with **RWG** basis functions

$$\underbrace{\begin{pmatrix} \eta_o \boldsymbol{T}_{k_o} + \eta_i \boldsymbol{T}_{k_i} & -(\boldsymbol{K}_{k_o} + \boldsymbol{K}_{k_i}) \\ \boldsymbol{K}_{k_o} + \boldsymbol{K}_{k_i} & \frac{1}{\eta_o}\boldsymbol{T}_{k_o} + \frac{1}{\eta_i}\boldsymbol{T}_{k_i} \end{pmatrix}}_{\boldsymbol{Z}} \underbrace{\begin{pmatrix} \boldsymbol{j} \\ \boldsymbol{m} \end{pmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{pmatrix} \boldsymbol{e} \\ \boldsymbol{h} \end{pmatrix}}_{\boldsymbol{b}}$$

$\boldsymbol{T}_k$ **EFIO** $\boldsymbol{K}_k$ **MFIO**
x_o, x_i **in/outside quantity**
$\boldsymbol{x}$ **unknown**
$\boldsymbol{b}$ **excitation**

- **Low Frequency Breakdown:** Originates from the **EFIO**



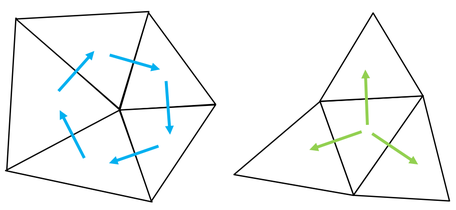
$$\boldsymbol{\mathcal{T}}_k = -\jmath k \boldsymbol{\mathcal{T}}_{A,k} - \jmath k^{-1} \boldsymbol{\mathcal{T}}_{\Phi,k}$$

Vanishes — Unbounded

Condition number $\kappa(\boldsymbol{\mathcal{T}}_k) = \frac{\sigma_{\max}(\boldsymbol{\mathcal{T}}_k)}{\sigma_{\min}(\boldsymbol{\mathcal{T}}_k)}$

→ $\kappa(\boldsymbol{\mathcal{T}}_k) \underset{k\to 0}{\to} \infty$ ⚠ **Slow** computation, **Inaccurate** solution

- **Loop-Star Decomposition**



Loop $\nabla \cdot \boldsymbol{f}_L = 0$ — **Star** $\nabla \cdot \boldsymbol{f}_S \neq 0$

Matrices $\boldsymbol{\Lambda}$ and $\boldsymbol{\Sigma}$ to pass from **Loop** and **Star** to RWG are then **rescaled separately**

→ $\boldsymbol{\mathcal{T}}_k(\boldsymbol{f}_L) = \boldsymbol{\mathcal{T}}_{A,k}(\boldsymbol{f}_L)$

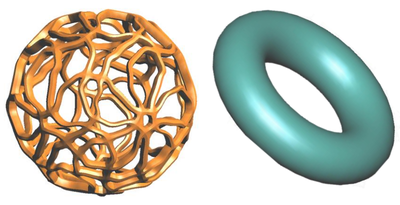
- **Quasi-Helmholtz Projectors**
  Can fulfill the same role with a **better conditioning**

$$\boldsymbol{P}^{\boldsymbol{\Sigma}} = \boldsymbol{\Sigma}\left(\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{\Sigma}\right)^{+}\boldsymbol{\Sigma}^{\mathrm{T}}$$
$$\boldsymbol{P}^{\boldsymbol{\Lambda}} = \mathbf{I} - \boldsymbol{P}^{\boldsymbol{\Sigma}}$$

Not sufficient for our conditions!

Future work

- **Extension to multiply connected geometries**
  A different, more intricate strategy is required for structures containing **holes** or **handles**, which introduce **global loops**.



Submitted and published works

- T.L. Chhim, S.B. Adrian, and F.P. Andriulli, "*On the Spectral Behavior and Normalization of a Resonance-Free and High-Frequency Stable Integral Equation*", 2018 IEEE APS/URSI, 8-13 July, Boston, USA, 2018.
- T.L. Chhim, J.E. Ortiz, L. Rahmouni, A. Merlini, and F.P. Andriulli, "*A Quasi-Helmholtz Projector Stabilized Full Wave Solver Encompassing the Eddy Current Regime*", ICEAA/IEEE APWC 2019, 9-13 September, Granada, Spain, 2019.

Adopted methodologies

- **Asymptotic Analysis:** With a **Loop-Star** decomposition → Breakdown when

$$\boldsymbol{Z}_{\mathrm{LS}} = \mathcal{O}\begin{pmatrix} \boldsymbol{\Lambda} & \boldsymbol{\Sigma} & \boldsymbol{\Lambda} & \boldsymbol{\Sigma} \\ \omega & \omega & \omega^2 & 1 \\ \omega & \omega + \frac{1}{\omega} + \frac{1}{\sigma} & 1 & 1 \\ \omega^2 & 1 & \sigma + \omega & \sigma + \omega \\ 1 & 1 & \sigma + \omega & \sigma + \omega + \frac{1}{\omega} \end{pmatrix} \begin{matrix} \boldsymbol{\Lambda} \\ \boldsymbol{\Sigma} \\ \boldsymbol{\Lambda} \\ \boldsymbol{\Sigma} \end{matrix}$$

$\omega \to 0$ **Low frequency**

$\sigma \to \infty$ **High conductivity**

Different regimes to treat

- **Quasi-Helmholtz Projectors**

$$\boldsymbol{Z}' = \begin{pmatrix} \boldsymbol{M}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{M}_2 \end{pmatrix} \boldsymbol{Z} \begin{pmatrix} \boldsymbol{M}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{M}_2 \end{pmatrix}$$

$$\boldsymbol{M}_1 = \alpha \boldsymbol{P}^{\boldsymbol{\Lambda}} + \mathrm{j}\,\beta \boldsymbol{P}^{\boldsymbol{\Sigma}}$$
$$\boldsymbol{M}_2 = \gamma \boldsymbol{P}^{\boldsymbol{\Lambda}} + \mathrm{j}\,\delta \boldsymbol{P}^{\boldsymbol{\Sigma}}$$

1. σ constant and $\omega \to 0$
2. ω constant and $\sigma \to \infty$
3. $\omega\sigma$ constant and $\omega \to 0$

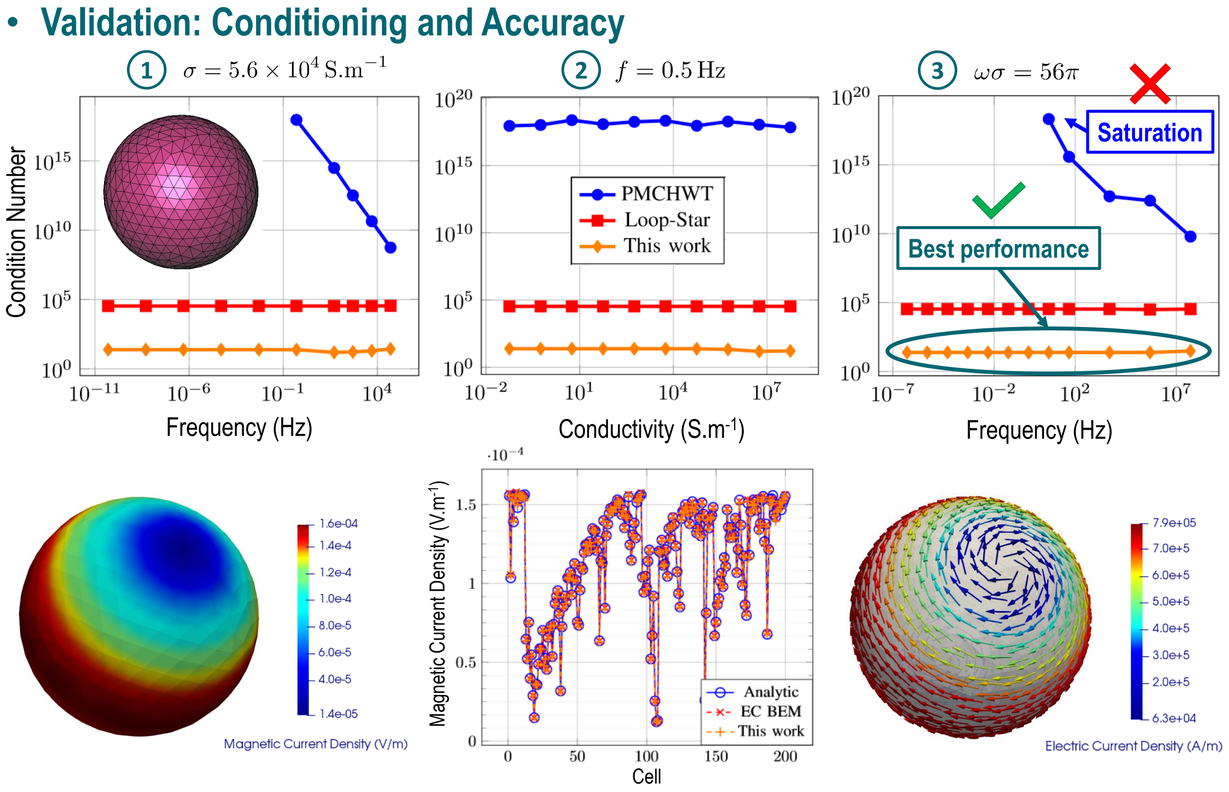
Novel contributions

- **Rescaling Factors**

Selection conditions:
1) All terms must be **bounded**
2) No **null spaces**

Regime	α	β	γ	δ
1	$1/\sqrt{\omega}$	$\sqrt{\omega}$	1	$\sqrt{\omega}$
2	1	1	$1/\sqrt{\sigma}$	$1/\sqrt{\sigma}$
3	$(\sigma/\omega)^{1/4}$	$(\omega/\sigma)^{1/4}$	$(\omega/\sigma)^{1/4}$	$(\omega/\sigma)^{1/4}$

- **Validation: Conditioning and Accuracy**



List of attended classes

- 01SFVRV – Metamaterials: Theory and multiphysics applications (03/2018, 20 hours)
- 01QTEIU – Data mining concepts and algorithms (03/2018, 20 hours)
- 01MMRRV – Advanced Computational Electromagnetics for Antenna Analysis and Design (03/2019, 20 hours)
- External (IMT Atlantique, France) – Fast Computing Solvers (01/2018, 20 hours)
- External (European School of Antennas, Italy) – Microwave Imaging and Diagnostics: Theory, Techniques, and Applications (03/2018, 30 hours)
- External (European School of Antennas, Italy) – Advanced Computational Electromagnetics (09/2018, 30 hours)
- 01SWPRV – Time management (11/2018, 2 hours)
- 02RHORV – The new Internet Society: entering the black-box of digital innovations (12/2018, 6 hours)
- 01SWQRV – Responsible research and innovation (12/2018, 5 hours)
- 01SYBRV – Research integrity (02/2019, 5 hours)
- 01SHMRV – Entrepreneurial Finance (03/2019, 5 hours)
- 02LWHRV – Communication (04/2019, 5 hours)
- 01RISRV – Public speaking (04/2019, 5 hours)

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