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XXXIV Cycle

# Numerical models for multi-wavelength mid-IR Quantum Cascade Lasers


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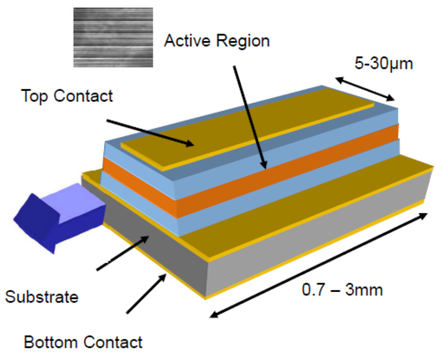
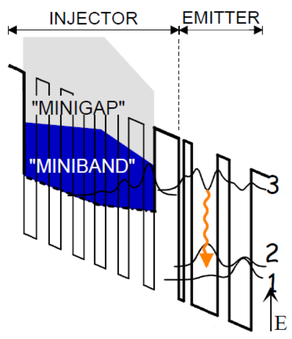
## Research context and motivation

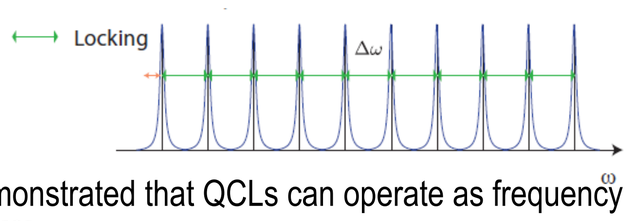
- **Quantum Cascade Lasers (QCLs).** Quantum Cascade Lasers are a particular class of semiconductor lasers, where the electronic transitions occur between confined states in the conduction band, which are named subbands. Therefore these transitions do not involve holes: **QCLs are unipolar devices** and transition energies are fully controlled by quantum confinement.




The emission spectral regions interested by QCLs are mid-IR (wavelength between 3 μm and 24 μm) and Terahertz (wavelength greater than 65 μm). Their structure is made of different stages, where each stage consists of **injector** and **emitter** regions, composed by several Quantum Wells about 10 nm wide. Sequential resonant tunneling and scattering between Longitudinal Optical(LO) phonons and electrons are other peculiar features of these devices. Ultra-fast carrier dynamics is achieved in QCLs, since the typical electron lifetime has the picosecond order of magnitude.

- **Optical frequency combs (OFC).** An Optical Frequency Comb is a set of equally spaced optical lines having constant phase difference and amplitude [1]. It has been recently demonstrated that QCLs can operate as frequency combs in both mid-IR and THz region [2].




- **Applications.** A particularly appealing application of OFCs is the so called dual comb spectroscopy. This technique is characterized by high resolution, high sensitivity and no moving parts and its use is especially interesting in the mid-infrared region, where most fundamental roto-vibrational absorption bands of light molecules can be found. Another interesting field of application for OFCs is the optical communication area where they may be used for the generation of sub-Thz signal for wireless networks.


[1] J.Faist et al., Nanophotonics, 5, 272-291 (2016).
[2] Hänsch, T. W. Nobel lecture: Passion for precision. Rev. Mod. Phys.78, 1297–1309 (2006)


## Addressed research questions/problems

- **Development of a semiconductor-like model.** The available models for the description of the multi-mode behaviour of QCLs are based on the theory of two or three levels laser and then incomplete. Our new approach exploits instead a more realistic phenomenological expression of the optical susceptibility which well reproduces the optical properties of the semiconductor active medium and leads to a set of Effective Semiconductor Maxwell-Bloch Equations (ESMBEs).
- **Accounting for a realistic Fabry-Perot configuration.** The laser cavity configuration which corresponds to real devices is the Fabry-Perot. The inclusion of this configuration into our model allows us to account for the formation of a carrier grating inside the laser cavity, due to the Spatial-Hole Burning, which influences the laser dynamics and provide for a more realistic description of the system, compared with the ring cavity configuration considered so far.
- **Reduction of the simulation time.** Since the complete description of the laser behaviour is based on a complex set of nonlinear partial differential equations (ESMBEs), the numerical simulations are computationally heavy. An effort to obtain a more simple mathematical approach is planed, in order to reduce the total simulation time.
- **Reproducing of the experimental results.** A tuning of the model parameters will possibly allow to reproduce the experimental results performed both in the mid-IR and THz region that show a peculiar behavior of the system, characterized by an alternance between locked and unlocked regimes, and the presence of a continuous wave emission just above the laser threshold.

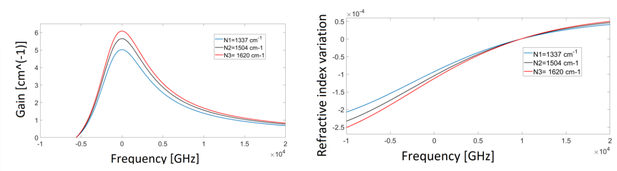
## Submitted and published works


- Silvestri, C., Columbo, L.L., Brambilla, M., and Gioannini, M., *"Effective Semiconductor Maxwell-Bloch Equations for a multi-mode Quantum Cascade Laser"*, presented at ESLW 2019, 27-28 September 2019, Cork, Ireland


## Adopted methodologies

### Optical Susceptibility

- alpha-factor ≠ 0
- Gain and refractive index dependence from carrier density
- Asymmetric profile of gain and refractive index as functions of the frequency.




### Effective Semiconductor Maxwell-Bloch Equations(ESMBEs)

$$\pm\frac{\partial E_0^{\pm}}{\partial z}+\frac{1}{v}\frac{\partial E_0^{\pm}}{\partial t}=-\frac{\alpha_L}{2}E_0^{\pm}+gP_0^{\pm}$$
$$\frac{\partial P_0^{\pm}}{\partial t}=\frac{\Gamma(1+i\alpha)}{\tau_d}\{-P_0^{\pm}-if_0\varepsilon_o\varepsilon_b(-1-i\alpha)(N_0E_0^{\pm}+N_1^{\pm}E_0^{\mp})\}$$
$$\frac{\partial N_0}{\partial t}=\frac{I}{eV}-\frac{N_0}{\tau_e}+\frac{i}{4\hbar}\{E_0^{+*}P_0^{+}+E_0^{-*}P_0^{-}-E_0^{+}P_0^{+*}-E_0^{-}P_0^{-*}\}$$
$$\frac{\partial N_1^{+}}{\partial t}=-\frac{N_1^{+}}{\tau_e}+\frac{i}{4\hbar}\{E_0^{-*}P_0^{+}-E_0^{+}P_0^{-*}\}$$

$f_0$=differential gain
$\omega_0$=reference frequency
n=refractive index
v=group velocity
Γ=adimensional coefficient related to the gain/dispersion bandwith
$\Gamma_C$=confinement factor
I=injection current
$\tau_d$=dephasing time
$\tau_e$=electron lifetime
$\alpha_L$=losses
α =alpha-factor
V=volume of the single stage
$N_p$=number of stages

$$g=\frac{-i\omega_0}{2\varepsilon_0 nc}N_p\Gamma_C$$

$E_0^{\pm}$= Electric field slowly varying envelope
$P_0^{\pm}$= Electric field slowly varying envelope
$N_0$= Carrier density at order 0
$N_1^{+}$=Carrier density due to SHB

### OFC indicators based on multimode dynamics

$$\mu_{P_q}=<P_q(t)>$$
$$\mu_{\Delta\Phi_q}=<\Delta\Phi_q(t)>$$
$$\sigma_{P_q}=\sqrt{<\left(P_q(t)-\mu_{P_q}\right)^2>}$$
$$\sigma_{\Delta\Phi_q}=\sqrt{<\left(\Delta\Phi_q(t)-\mu_{\Delta\Phi_q}\right)^2>}$$

$$M_{\sigma_{P_q}}=\frac{1}{N_{10dB}}\sum_{q=1}^{N_{10dB}}\sigma_{P_q}$$

Low value of $M_{\sigma P_q}$ corresponds to low amplitude noise

$$M_{\Delta\Phi_q}=\frac{1}{N_{10dB}}\sum_{q=1}^{N_{10dB}}\sigma_{\Delta\Phi_q}$$

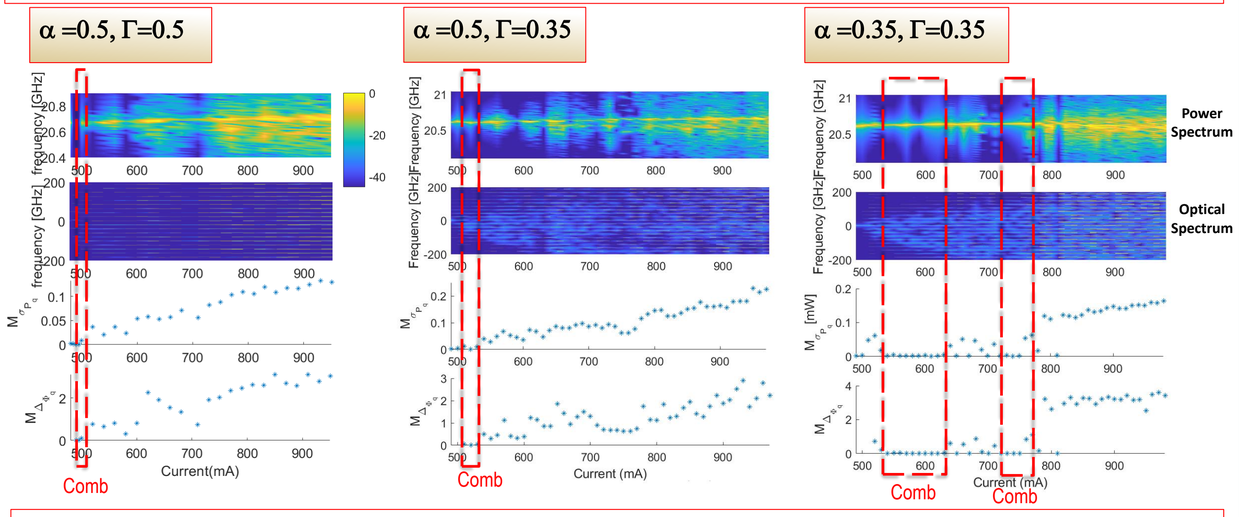
Low value of $M_{\Delta\Phi_q}$ corresponds to low phase noise

Differential equations have been integrated by using a finite differences scheme, discretizing both in time and space. Numerical simulations have been performed exploiting several calculation servers, which share a common virtualization/storage infrastructure.

## Novel contributions

### Simulation results

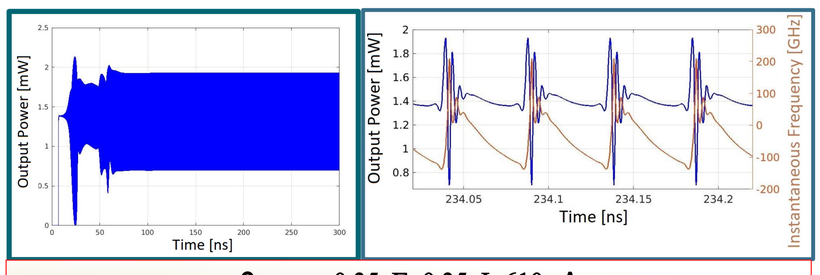
Results of numerical simulations of the multi-mode dynamics of a typical Quantum Cascade Laser emitting in the mid-IR region are shown.




Power Spectrum, Optical Spectrum and OFC indicators trend by sweeping the bias current, for three different combination of the alpha-factor and the gain/dispersion bandwidth.

| |
|---|
| L=2mm |
| τd =0.1 ps |
| τe= 1 ps |
| f0= 1.1*10$^{-7}$ μm^3 |
| R=0.3 |

Systematic simulations show that the extension of the OFC regime decreases by increasing the alpha-factor and the gain/dispersion bandwidth.




Case α =0.35, Γ=0.35: I=610mA
Evidence of linear chirp and regular dynamics, in agreement with experimental results

Alternance between regular (OFC) and irregular regimes by sweeping the bias current.

## Future work

- Reduction of the ESMBEs to two master equations for the forward and backward fields, which will unify the QCL lasers to the family of physical systems described by modified CGLE systems. This will allow to make the role of critical parameters in influencing the self-locking mechanism and to reduce the simulation time.
- Reproduce recent experimental results on direct modulation of QCL, achieving an increase of the extension of the comb regime and bandwidth, in collaboration with the Laboratoire Matériaux et Phénomènes Quantique (MPQ), Université Denis Diderot- Paris VII.

## List of attended classes

- 01NVOPE – CAD of semiconductor devices and processes (19/2/2019, 6 CFU)
- 02LWHRV – Communication (9/7/2019, 1 CFU)
- 01UGERO – Materials by design - How structure meets function (3/7/2019, 3CFU)
- 01QFDRV – Photonics: a key enabling technology for engineering applications(24/7/2019, 5 CFU)
- 01SFURV – Programmazione scientifica avanzata in Matlab (27/6/2019, 4 CFU)
- 01SWPRV – Time management (2/1/2019, 1 CFU)
- 01NOPOQ – Photonic Devices (October 2018-January 2019, exam to be taken)

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