ScuDo

Scuola di Dottorato ~ Doctoral School

WHAT YOU ARE, TAKES YOU FAR

XXXIV Cycle

Nonlinear system identification in Sobolev spaces

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Research context and motivation

- Mathematical modelling of dynamic systems is the very first step for studying, analyzing, and hoping to control a physical system. Accurate dynamic systems models are needed by industries or by academia in many engineering or non-engineering fields, for purposes such as prediction, simulation, control design, decision making, fault detection, etc.
- System identification aims at building mathematical models of dynamic systems using a finite set of noise-corrupted input-output experimental data and prior information on the unknown system and on the noise.
- Consider a nonlinear discrete-time system represented in input-output regression form:
$$y_{k+1} = f_o(x_k) + \xi_{k+1}, \qquad x_k = \left(y_k, \cdots, y_{k-m_u}, u_k, \cdots, u_{k-m_u}\right)$$
and suppose the function f_o unknown, but a set of noisy measurements data available:
$$D = \{\tilde{y}_{k+1}, \tilde{x}_k\}_{k=1}^{L}, \qquad \tilde{y}_{k+1} = f_o(\tilde{x}_k) + d_k$$
- The system identification problem can be seen as a function approximation problem, aiming at finding an approximating function $\hat{f} \cong f_o$.
- A standard approach to system identification is to adopt a parametrized model structure for $\hat{f}$ (e.g. NARX or NOE) and to estimate the involved parameters by minimizing the model prediction or simulation error. These models just aims to minimize the one-step prediction or the simulation error, without really trying to capture the relation between the single components of the regressor x_k and the output y_{k+1}.
- The availability of an accurate multi-step prediction model is of paramount importance in Nonlinear Model Predictive Control. At every time k, given the input and output regressors, the model should correctly describe the variations of the predicted output $\hat{y}_{k+\tau}, \tau \geq 1$, due to the variations of the command input sequence.
- Our key observation is that function derivatives are crucial to determine the relation between each component of x_k and y_{k+1} in the identification problem, and the variations of the predicted output, due to the variations of the command input sequence in the NMPC context. Models identified by accounting for the derivatives are more accurate and reliable than the traditional ones, based on plain function approximation, and can provide a better performance in several tasks, such as multi-step prediction, simulation and Nonlinear Model Predictive Control.

Addressed research questions/problems

- We consider the problem of identifying from a set of experimental data an estimate $\hat{f}$ of f_o, such that, not only $\hat{f}$ approximates f_o, but also the first derivatives of $\hat{f}$ approximate the first derivatives of f_o.
- In the literature, we found only a few works proposing methods for approximating from data a function and its derivatives. Only a limited number of works carry out a theoretical analysis about the approximation properties of these techniques, and the provided results often just prove existence of the required approximating function. Another lack we observed is that the proposed techniques allow the identification of a model, but they do not provide a description of the uncertainty associated with this model and its predictions. Moreover, the works we found typically assume that the function derivative samples are available, but this may not be true in a real application.
- Being S_{1p} the Sobolev space, consider a function $f_o \in S_{1p}$, defined by
$$z = f_o(x), \qquad f_o\colon X \subset \mathbb{R}^{n_x} \to \mathbb{R}$$
and suppose that f_o is unknown, but a set of noise-corrupted data is available:
$$D = \left\{\tilde{x}_k, \{\tilde{z}_k^i\}_{i=0}^{n_x}\right\}_{k=1}^{L}, \qquad \tilde{z}_k^i = f_o^{(i)}(\tilde{x}_k) + d_k^i, \qquad i = 0, \dots, n_x, k = 1, \dots, L$$
where with $i > 0$ are denoted the ith partial derivative or its measurements. Furthermore, the noise sequences d^i are assumed unknown but bounded:
$$\left\|d^i\right\|_q \leq \mu^i, \qquad 0 \leq \mu^i \leq \infty$$
- Considering the following identification error $e(\hat{f}) \doteq \left\|f_o - \hat{f}\right\|_{S_{1p}}$, we are looking for an approximation of f_o in the S_{1p} Sobolev space.
- A parametrized structure is adopted for $\hat{f}$: $\hat{f}(x) = \sum_{j=1}^{N} a_i \phi_j(x)$.
- The considered problem can be stated as follows.
From the dataset D, identify an approximation $\hat{f}$ of the adopted structure: (1) giving a small Sobolev identification error; (2) equipped with guaranteed uncertainty bounds on the unknown function f_o and its derivatives.

Submitted and published works

- Novara, C. , Nicolì, A., and Calafiore, G.C., "*Nonlinear system identification in Sobolev spaces*", to be submitted

Novel contributions

- The idea of approximating the unknown function first-order derivatives, in addition to the function itself, which improves the identified model quality and the accuracy of the predictions with respect to traditional models, based on plain function approximation.
- Inclusion of a theoretical optimality analysis, showing that models obtained using the approach enjoy suitable optimality properties in Sobolev spaces.
- The derivation of tight uncertainty bounds on the unknown function and its derivatives, quantifying the modeling error and the prediction uncertainty.
- A technique for estimating partial derivatives outputs from the function input-output data is also provided, in case of unavailability of their measurements in real world applications.

Adopted methodologies

- The approximating function is found by solving one of the two equivalent convex optimization problems:

$$\tilde{z}^i \doteq \begin{bmatrix} \tilde{z}_1^i \\ \vdots \\ \tilde{z}_L^i \end{bmatrix}, \quad \Phi^i \doteq \begin{bmatrix} \phi_1^{(i)}(\tilde{x}_1) & \cdots & \phi_N^{(i)}(\tilde{x}_1) \\ \vdots & \ddots & \vdots \\ \phi_1^{(i)}(\tilde{x}_L) & \cdots & \phi_N^{(i)}(\tilde{x}_L) \end{bmatrix}$$

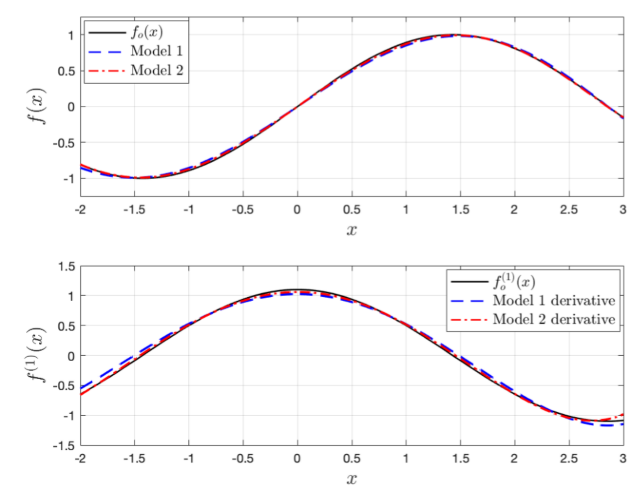
$$a = \arg\min_{\alpha \in \mathbb{R}^N} \|\alpha\|_r \qquad\qquad a = \arg\min_{\alpha \in \mathbb{R}^N} \sum_{i=0}^{n_x} \lambda^i \left\|\tilde{z}^i - \Phi^i \alpha\right\|_q^2 + \Lambda\|\alpha\|_r$$

$$s.t. \left\|\tilde{z}^i - \Phi^i \alpha\right\|_q \leq \mu^i \quad, i = 0, \dots, n_x$$

- Optimality analysis and results already obtained in the Set Membership framework by the research group regarding approximation in Banach $\mathcal{L}_p$ spaces, were extended to Sobolev spaces, showing that the derived approximating functions are almost-optimal in two cases: (1) when is assumed f_o to belong to a Sobolev space; (2) when the further assumption of Lipschitz continuity of the derivatives of the residue function $f_o - \hat{f}$ is made.
- Tight uncertainty bounds for f_o and its derivatives are derived as an extension of the ones already obtained by the research group in Banach $\mathcal{L}_p$ spaces, to the case where the bounds are derived not only for the function but also for its first-order derivatives.

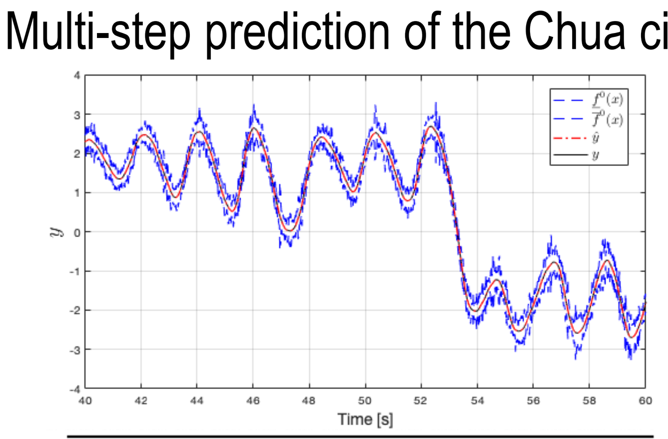
Numerical examples

Univariate function approximation



Estimations	RMSE	RMSE$^{(1)}$
Model 1	2.54e-02	5.86e-02
Model 2	1.19e-02	2.57e-02

Multi-step prediction of the Chua circuit



Predictors	RMSE$_3$	RMSE$_5$	RMSE$_7$
P1_NOD	6.18e-02	1.03e-01	1.44e-01
P1_D	5.59e-03	1.29e-02	2.33e-02
P1_ED	3.56e-03	1.01e-02	2.09e-02
PK_NOD	5.96e-02	9.90e-02	1.39e-01
PK_ED	6.22e-04	1.07e-03	2.17e-03

Future work

- Adopting the proposed identification approach in the data-driven Nonlinear Model Predictive Control in automotive and biomedical applications.
- Comparing the control results obtained by adopting our approach with the ones obtained via classical (data-driven or not) NMPC versions.
- Theoretical analysis of the stability properties of the closed-loop system.

List of attended classes

- 01LCPRV – Experimental modeling: costruzione di modelli da dati sperimentali (04/02/2019, 6)
- 01TEVRV – Deep learning (didattica di eccellenza) (04/06/2019, 6)
- 01SFURV – Programmazione scientifica avanzata in matlab (27/06/2019, 4)
- 01SWPRV – Time management (18/03/2019, 1)
- 01RISRV – Public speaking (29/03/2019, 1)
- 02LWHRV – Communication (05/05/2019, 1)
- Other unregistered courses

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