A multi-sample detection algorithm for channels affected by phase noise

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Research context and motivation

- The phase noise, a random fluctuation in the carrier phase caused by the instability of local oscillators, plays a major role in the degradation of most communication systems' performance. To give some examples, we can name:
  - IoT applications, where user terminals generally embed a low-cost local oscillator [1];
  - orthogonal frequency-division multiplexing (OFDM) systems, quite sensitive to phase noise [2];
  - low-bit-rate transmissions, where the phase noise can vary up to several degrees during a symbol period, resulting in a phase noise spectral occupancy wider than the useful signal bandwidth [3]. In this framework, future Deep Space missions (such as Ice Giants, Lagrange, and ODINUS) will target very distant regions in outer space. Successful closure of the communication link is possible only by resorting to low bit rates, thus facing low signal-to-noise ratios, Doppler impact, phase noise, and other impairments.
- The research focuses on studying advanced synchronization and coding techniques to cope with phase-noise limited channels.

Addressed research questions/problems

- We consider the transmission over an AWGN channel affected by phase noise. At the reception, the transmitted signal $x(t)=\sum_k c_k p(t-KT)$ has complex base-band expression

$$y(t)=x(t)e^{j\,\theta(t)}+w(t), \tag{1}$$

with $p(t)$ the shaping pulse, T the symbol time, $\{c_k\}$ the sequence of information symbols belonging to the M-ary complex-valued constellation, $w(t)$ the complex-valued white Gaussian noise having power spectral density N_0, and $\theta(t)$ the phase noise modelled as a Wiener random process.

- Most of the studies [4] adopt a discrete symbol-level channel model that reduces (1) to

$$\widetilde{y_k}=c_k e^{j\theta_k}+w_k, \tag{2}$$

where $\theta_k=\theta(kT)$ is the phase noise sampled at multiples of the symbol time and having variance σ_Δ^2.

Nevertheless, this approximation implies the equivalence of $\{\widetilde{y_k}\}$ to the matched filter (MF) output and holds while the phase noise process varies "slow" with respect to T. On the contrary, when σ_Δ^2 is large, this assumption no longer holds.

- To tackle this problem, we adopted a channel model based on the oversampling method described in [5]. For an oversampling factor η, $y(t)$ is assumed to pass through an ideal low-pass filter (LPF) with bandwidth η/T and then sampled with an interval of T/η. Thus, for values of η large enough, it holds

$$\widetilde{y_k}=\sum_i c_i\, p_{k-i\eta}e^{j\theta_k}+w_k, \tag{3}$$

where $p_k=p(kT/\eta)$, $\theta_k=\theta(kT/\eta)$ with variance $\sigma_\Delta^2\eta$, and w_k having variance $\mathrm{N}_0\eta$.

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List of attended classes

- 01UNXRV – Thinking out of the box (20-12-2021, 1)
- 01UNYRV – Personal branding (21-12-2021, 1)
- 01SWPRV – Time management (22-12-2021, 1)
- 02LWHRV – Communication (26-12-2021, 1)
- Digital Communications (2-2-2022, 72 h)
- 01RISRV – Public speaking (19-4-2022, 1)
- 02SFURV – Programmazione scientifica avanzata in Matlab (26-04-2022, 6)
- 01TSGKG – The Monte Carlo method (6-5-2022, 6)
- Standardization Training Course (21-6-2022, 31 h)

Adopted methodologies

- We derived the multi-sample algorithm in the BCJR form using the factor graph in Fig. 1 and the sum-product algorithm [6] under the simplifying assumption of a symbol time-limited shaping pulse. The channel model in (3) becomes:

$$\widetilde{y_k}=c_{\lfloor k/\eta\rfloor}e^{j\theta_k}+w_k, \tag{4}$$

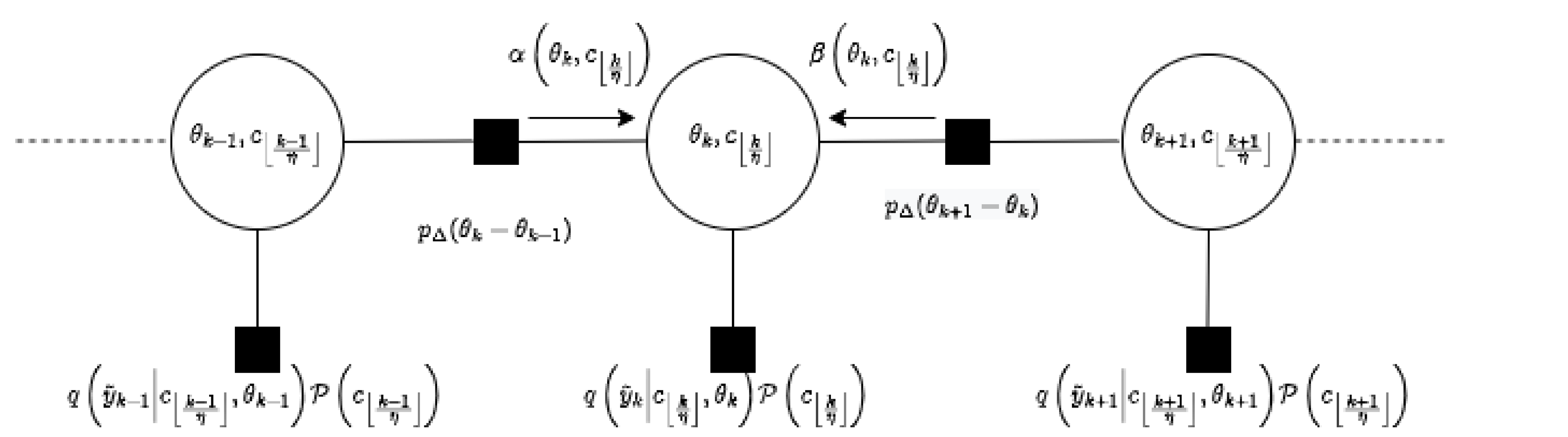

Figure 1: Factor graph

- $\alpha\left(\theta_k, c_{\left\lfloor\frac{k}{\eta}\right\rfloor}\right)$ and $\beta\left(\theta_k, c_{\left\lfloor\frac{k}{\eta}\right\rfloor}\right)$, the forward and the backward recursion, respectively
- $p_{\Delta(\theta_k-\theta_{k-1})}$ a wrapped Gaussian probability density function (pdf)
- $q(\tilde{y}_k|c_{\lfloor k/\eta\rfloor},\theta_k)$ is a Gaussian pdf with mean $c_{\lfloor k/\eta\rfloor}e^{j\theta_k}$ and variance $N_0\eta$
- $\mathcal{P}(c_{\lfloor k/\eta\rfloor})$ takes values $P(c_{\lfloor k/\eta\rfloor})$, namely, the probability of the information symbol $c_{\lfloor k/\eta\rfloor}$, if $(k+1)$ is multiple of η, or 1 otherwise.

Novel contributions

- We derived a detection algorithm based on a multi-sample receiver and in the case of a time-limited shaping pulse.
- The achievable information rate (AIR) (Fig. 2) proves the performance enhancement of the proposed algorithm, compared to any detection algorithm based on a symbol-level channel model.
- Fig. 3 shows the comparison between the classical BCJR and the multi-sample BCJR FER for a QPSK modulation with an LDPC (32400,64800) and affected by a phase noise with $\sigma_\Delta=28$ deg.

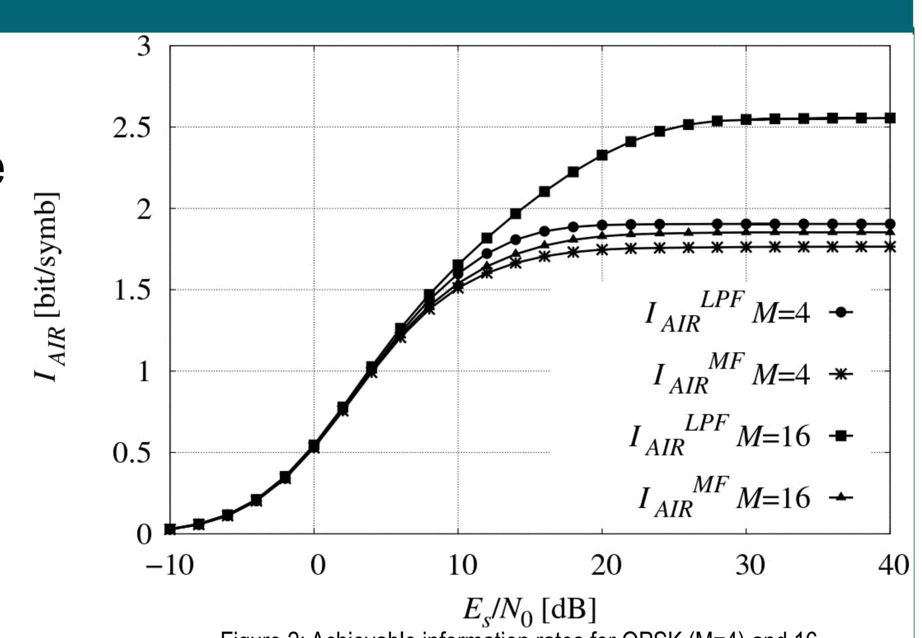

Figure 2: Achievable information rates for QPSK (M=4) and 16-PSK (M=16) constellation, with σ_Δ = 28 deg.

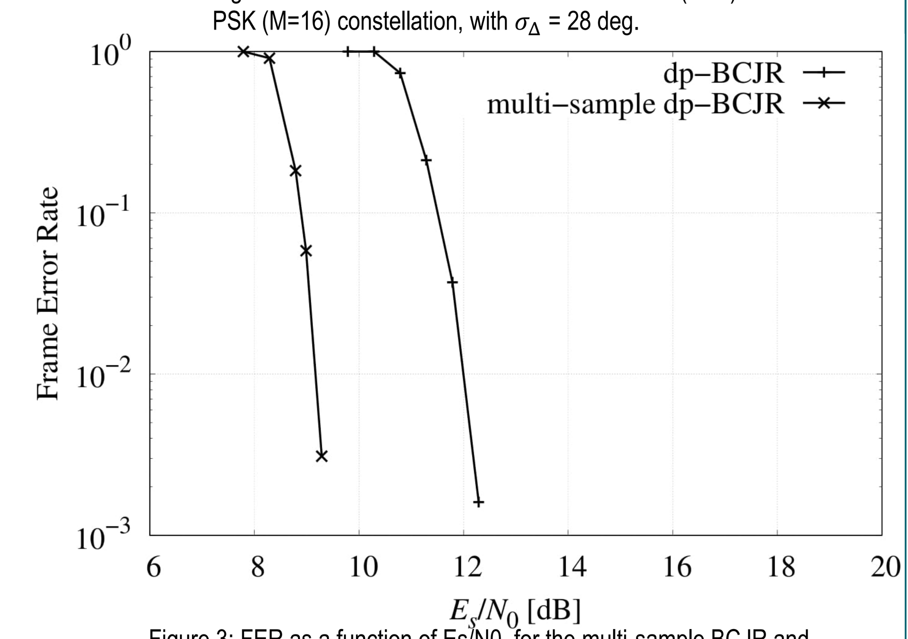

Figure 3: FER as a function of Es/N0, for the multi-sample BCJR and the classical BCJR with QPSK, when σ_Δ = 28 deg.

Future work

- The complexity of the multi-sample BCJR scales as $\mathcal{O}(NML^2\eta)$, thus becoming impractical for large constellations. Thus, the simplification of the problem could go in the direction of using canonical distributions. Such as:
  - Tikhonov pdf [7],
  - Expectation propagation [8]
- Approximate the phase noise probability distribution adopted by the multi-sample BCJR as the one that maximizes the achievable information rate, following the technique in [9]
- Once the detection algorithm will be tractable, it would be useful to re-design the error correcting codes to make them suitable for phase noise channels and thus achieving the rates predicted by the AIR [10].

Submitted and published works

- Ripani B., Modenini A., Garello R., Maiolini Capez G., and Montorsi G., "*On the use of Pseudo-Noise Ranging with high-rate spectrally-efficient modulations*", Proceedings of the 16th International Conference on Space Operations, Virtual 2021,
- B. Ripani, A. Modenini and G. Montorsi, "On the use of PN Ranging with High-rate Spectrally-efficient Modulations in Satellite Payload Telemetry Links," in IEEE Transactions on Aerospace and Electronic Systems, DOI: 10.1109/TAES.2022.3171210.
- Ripani B., Modenini A, and Montorsi G, "A multi-sample discrete-phase BCJR algorithm for phase noise channels", Global Communications Conference (GLOBECOM), Rio de Janeiro, Brazil, 2022.

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