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XXXVII Cycle

High-order and data driven numerical methods modelling scattering and propagation in computational electromagnetics

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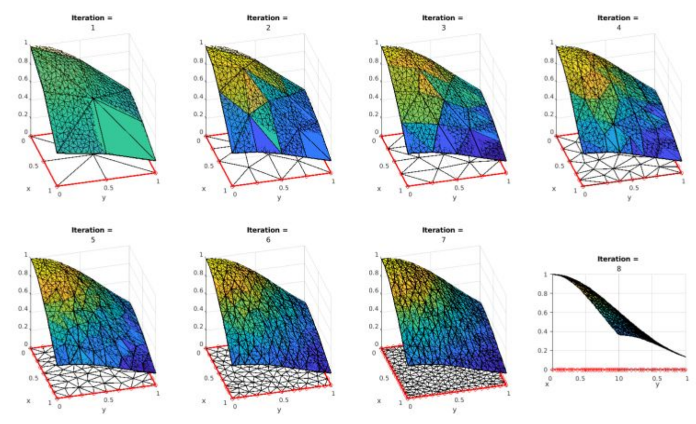
Research context and motivation

- Numerical simulations constitute a critical design process in fundamental sciences and engineering; as a result the development of rigorous and efficient **numerical methods** lies at the core of academic and industrial research interest. In applied electromagnetics they are distinctly powerful in complex 2 and 3D scattering and propagation problems. Those models refer to the boundary eigenvalued problems (BEVPs) based onto the Fourier-domain **Helmholtz curl-curl** (elliptic) equation in abstract variational formulation

find $(\boldsymbol{E}, \omega) \in H(\text{curl}, \Omega) \times \mathbb{R}^+$ s.t. $(\boldsymbol{\mu}^{-1}\nabla\times\boldsymbol{E}, \nabla\times\boldsymbol{T}) - \omega^2(\boldsymbol{\varepsilon}_{eq}\boldsymbol{E}, \boldsymbol{T}) = (\boldsymbol{f}, \boldsymbol{T}) \quad \forall\, \boldsymbol{T} \in H(\text{curl}, \Omega)$

- In the vast and heterogeneous landscape of numerical techniques in computational electromagnetics (CEM), **interpolatory-based finite methods** (FEM and BEM) established themselves as the most stable and reliable algorithms since their first successful application to the waveguide analysis [1]. This later culminated with the seminal formalisation [2] of **vector-valued subspaces**



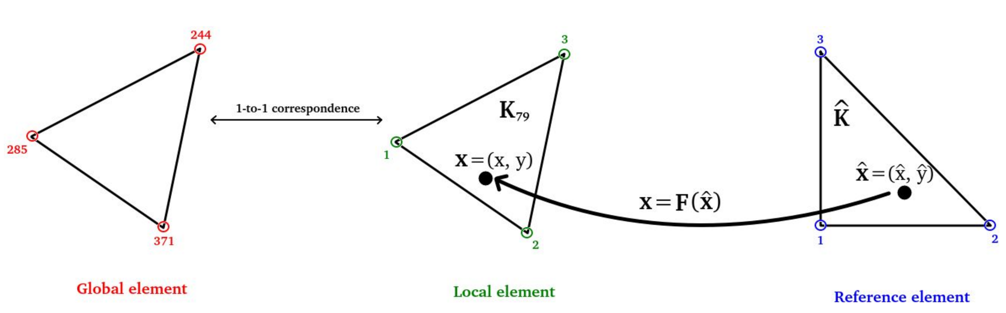
- Nowadays, widespread closed-source CAD/CAE softwares s.a. Ansys (*HFSS*), Altair (*Feko*) and 3DS (*CST Microwave Studio*) only implement **lowest-order elements** in their solvers, with no specialised basis functions, significantly limiting the *(hp)-refinement* of the numerical simulations and increasing the exploitation of resources during the design.

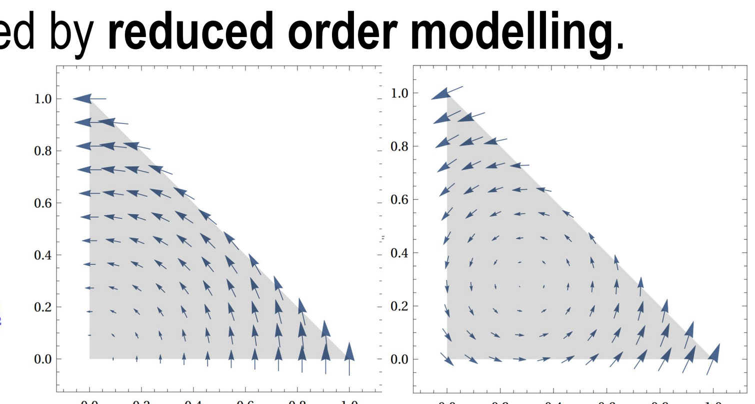
Addressed research questions/problems

- Following the variational approach introduced by Ciarlet's axiomatic definition of finite element, the curl and div conformity of H(curl, Ω) and H(div, Ω) edge elements at the lowest order has been possible thanks to the adoption [2,3] of Piola's transformation (covariant and contravariant respectively).
- The rigorous description and implementation of **arbitrary order edge elements** however proved to be somewhat cumbersome and required a non-insignificant amount of geometrical interpretation [4] to be accomplished.

$\varphi(\xi) = \prod_{n=1} R_n(p, \xi_n)\,, \quad R_n(p, \xi) = (n!)^{-1} \prod_{k=0}^{j-1} (p\xi - k)\,, \quad \boldsymbol{\varphi}(\xi) = (\boldsymbol{\varphi} \circ \boldsymbol{F}^{(\mathcal{K})})(\mathbf{x}) \in H(\text{curl}, \mathcal{K})$

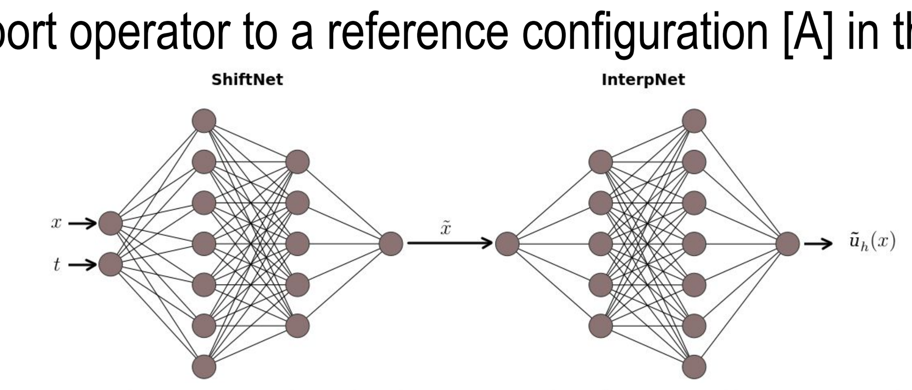
- Real-world problems in electrical engineering (radar, optics, antennas etc...) simultaneously demand the precise handling of the fields' singular behaviour, i.e. **high order modelling** of material and geometrical **singularities** [5], while performing reasonably fast simulations of parametric full-wave models, which is later addressed by **reduced order modelling**.



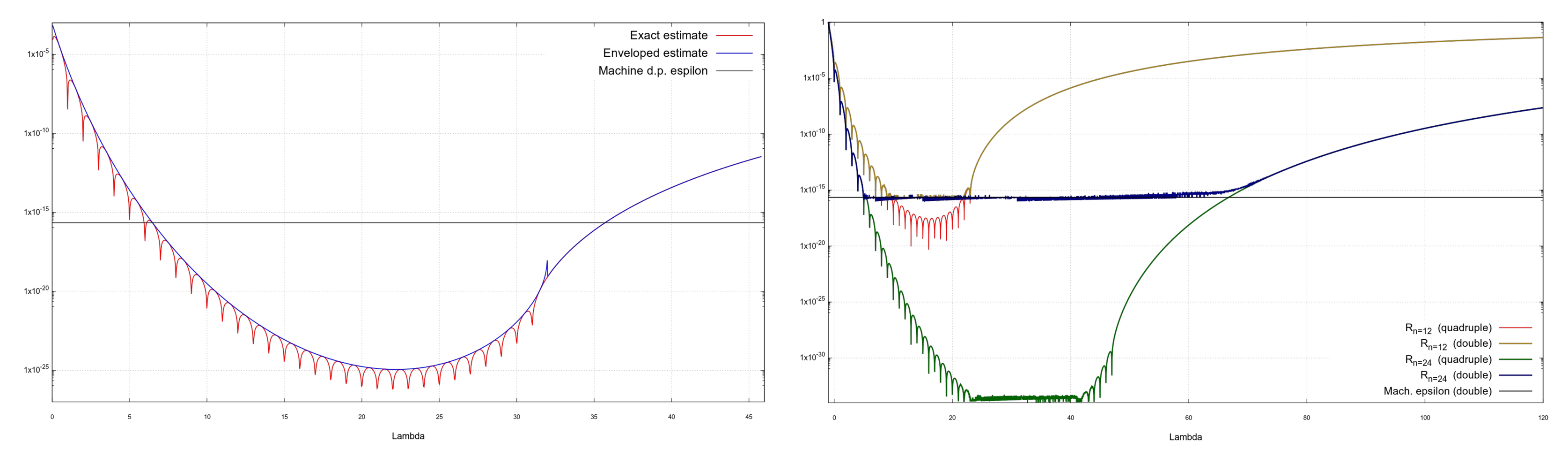


Novel contributions

- Well-established ROMs in fluid mechanics and electromagnetism (s.a. reduced basis and POD) represent a powerful tool in the hands of design engineers. **Hyperbolic equations** are infamously characterised by **slow Kolmogorov N-width decay** limiting the speed-up for the computation of the low-rank solution space. Several machine learning techniques have been adopted to find non-intrusive, non-linear transformations between manifolds; there is in fact a substantial advantage in exploiting the dominance of the advection term. **Deep neural networks** have been exploited to detect such optimal mappings and reconstruct a transport operator to a reference configuration [A] in the full-order model.



- In any variational numerical method the weak form dictates the need for computing precisely and efficiently the **integrals of the shape functions** in the linear and bilinear forms. This issue is traditionally addressed by **numerical quadrature** scheme based on Gaussian formulas. Those prove to be unsatisfactory for **generalised/singular polynomial integrands** which require a careful, ad-hoc recollocation of the nodes and weights of the formula [B] to better suit the singularity.



Adopted methodologies

- While **numerical analysis** is at the core of the research (especially FVMs, POD-Galerkin, Gauss-Legendre quadrature formulae and Lagrangian interpolation) important results in **complex** and **functional analysis** have also been used in the design of the algorithms.
- Distinct frameworks of **statistical machine learning** have been adopted to the case of scientific computing albeit in relatively simple configurations (compared to current trends).

Future work

- On the high-order side, **pre-assembled stiffness matrix VFEM** is currently being implemented with the integral contribution being computed with infinite (exact) precision.
- On the reduced-order side, **RFIDs/Metamaterials analysis** will be carried with ML-POD.

References

[1] Silvester, P., "*Finite-Element solution of homogeneous waveguide problems*", Alta Frequenza, Vol. 38, 1969.
[2] Nédélec, J-C., "*Mixed finite elements in $\mathbb{R}^3$* ", Numerische Mathematik, Vol. 35, 1980.
[3] Brezzi, F., Fortin, M., "Mixed and hybrid finite element methods", Springer Series in Computational Mathematics, Vol. 15, 1991.
[4] Graglia, R., Wilton, D., Peterson, A., "*Higher order interpolatory vector bases for computational electromagnetics*", IEEE Transactions on Antennas and Propagation, Vol. 45, Issue No. 3, 1997.
[5] Graglia, R., Lombardi, G., "*Singular higher order complete vector bases for finite methods*", IEEE Transactions on Antennas and Propagation, Vol. 52, Issue No. 7, 2004.

Submitted and published works

[A] Papapicco, D., Demo, N., Girfoglio, M., Stabile, G., and Rozza, G., "*The neural network shifted-proper orthogonal decomposition: a machine learning approach for non-linear reduction of hyperbolic equations*", Computer Methods in Applied Mechanics and Engineering, Vol. 392, Article No. 114687, **Published on 15/03/2022**.
[B] Lombardi, G., Papapicco, D., "*Algorithm XXX: QUASIMONT – A C++ monomial transformation quadrature rule for high-precision integration of singular and generalised polynomials*", ACM Transactions on Mathematical Software, **Submitted on 03/04/2022**.

List of attended classes

- XXIV Stage Scuola Nazionale Dottorandi di Elettrotecnica "Gasparini" (24/01/22, 24 hours)
- 01SFVRV – Metamaterials: theory and multiphysics applications (08/04/22, 20 hours)
- 01DOBRV – Mathematical-physical theory of electromagnetism (06/06/22, 15 hours)
- 01NVEOQ – Radiating electromagnetic systems (30/06/22, 40 hours)
- 01UJDRV – Integral operators and fast solvers (29/07/22, 21 hours)
- 01RISRV – Public speaking (21/01/22, 5 hours)
- 01UNYRV – Personal branding (21/01/22, 1 hour)
- 01UNXRV – Thinking out of the box (21/01/22, 1 hour)
- 02LWHRV – Communication (24/01/22, 5 hours)
- 01DOCRV – The Hitchhiker's guide to the academic galaxy (16/06/22, 20 hours)

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