Analysis, planning and control of complex redundant manipulators

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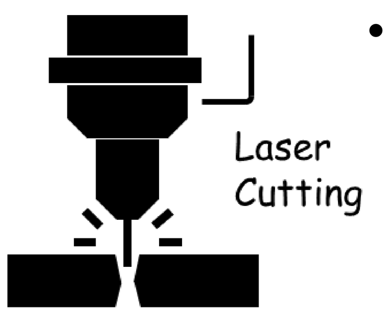
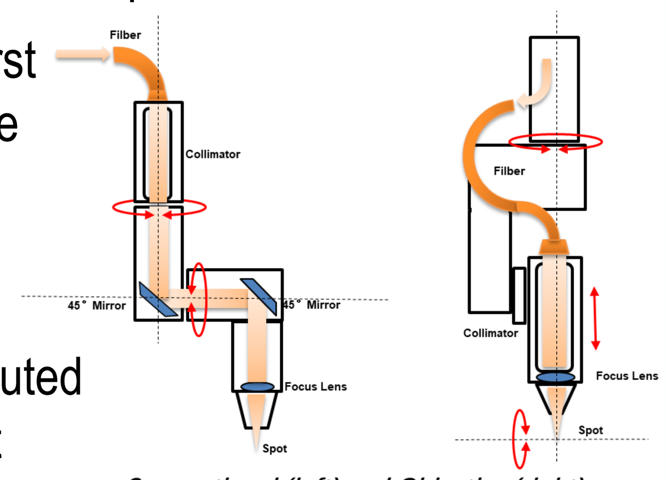
Research context and motivation



- An innovative redundant manipulator structure for 3D laser cutting of car-bodies is the core of a joint research project between Politecnico di Torino and EFORT. The goal is to improve the speed and performances of the manipulator in tracking the cutting trajectory of complex surfaces in 3D space.
- To overcome the shortcomings of the conventional mechanical wrists with offset, the novel structure is formed by a zero-offset wrist and a prismatic axis. The intrinsic redundancy can be exploited to compute a better path while tracking high curvature trajectories. But redundant axes management must comply with cogent technological requirements, so a constraint-based optimization algorithm must be used to accomplish the task.
- Optimization process is divided into two phases: in the first one the target points are optimized in the operating space through an off-line programming and simulation tool that performs a **workspace and reachability analysis, a singularity optimization** and **collision detection**. In the second phase the **optimization algorithm** is executed to compute the best trajectory program, before sending it to the real-time CNC for the production.



Conventional (left) and Objective (right) 3D Laser Cutting Head

Addressed research questions/problems

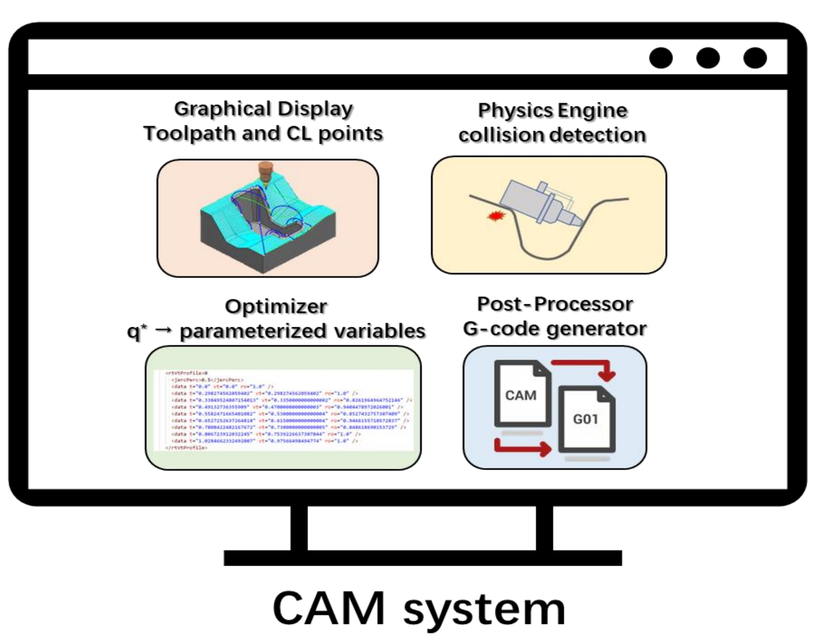
- Specific manipulator **kinematics modelling** and **workspace** analysis.
- The techniques of numerical optimization and multiple criteria of modeling the optimization with single output are studied.

$$\max \sum_{k=1}^{3} \dot{x}_k^2 < V_{max}^2$$

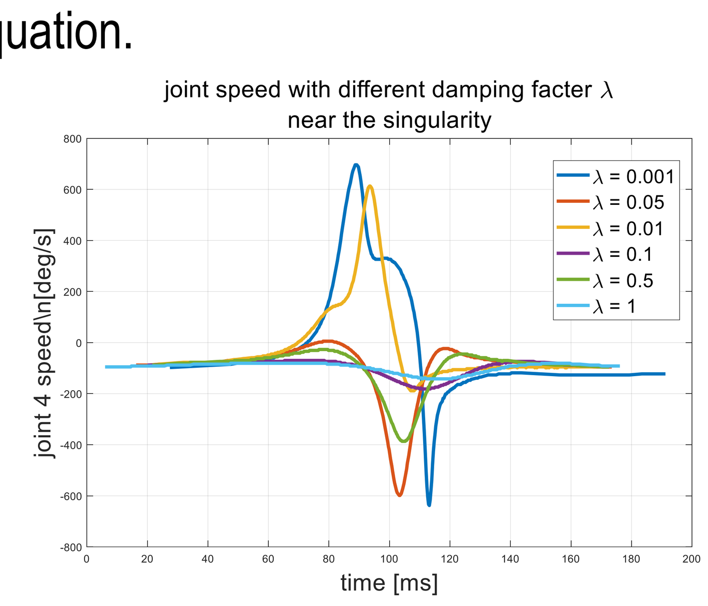
$$s.t.\ \dot{x} = \begin{bmatrix} \dot{x}_x \\ \dot{x}_y \\ \dot{x}_z \\ \dot{x}_a \\ \dot{x}_b \\ \dot{x}_c \end{bmatrix}, q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_{Aux} \end{bmatrix} = \begin{bmatrix} q_1' + \dot{q}_1 \\ q_2' + \dot{q}_2 \\ q_3' + \dot{q}_3 \\ q_4' + \dot{q}_4 \\ q_5' + \dot{q}_5 \\ q_{Aux}' + \dot{q}_{Aux} \end{bmatrix},$$

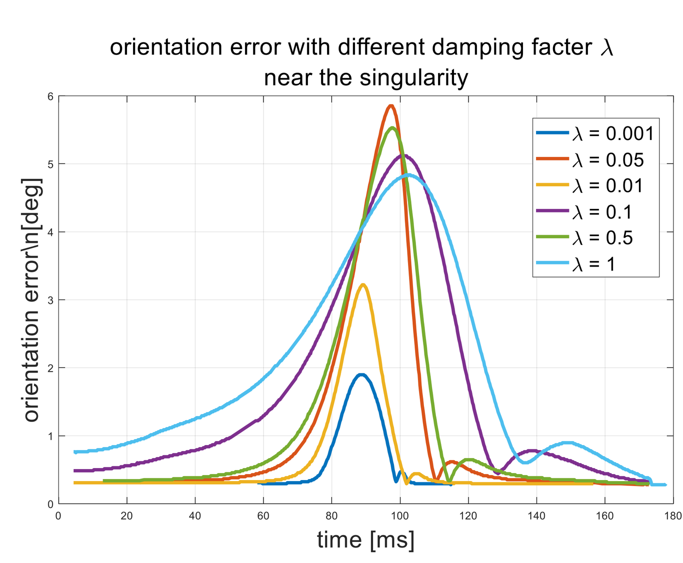
$$x = f(u),$$
$$\dot{q} = J^{-1}(\dot{x}),$$
$$g(q_4, q_5) < S_{max},$$
$$\forall q_k, \dot{q}_k, \ddot{q}_k \in P$$

$x, \dot{x} \in \mathbb{R}^6 \rightarrow$ Interpolated Cartesian point and velocity
$q_k, \dot{q}_k, \ddot{q}_k \in \mathbb{R}^6 \rightarrow$ computed axis position, velocity, acceleration data set in joint space
$q_{Aux} \in \mathbb{R} \rightarrow$ redundant prismatic joint value
$V_{max} \in \mathbb{R} \rightarrow$ max tangential speed depending on laser cutting technique
$P \in \mathbb{R}^{3 \times 6} \rightarrow$ actual limits of configuration of joint actuator
$f(u) \rightarrow$ geometric trajectory interpolation with input argument u abscissa curvilinear
$J^{-1}(\dot{x}) \rightarrow$ Jacobian inverse matrix to map corresponding differential joint value
$g(q_4, q_5) \rightarrow$ function to compute the angular difference between the approaching verso and CL z-axis, while S_{max} is the technical max angle



- The degenerate wrist Jacobian matrix is avoided by adding the damping factor near the singularity, the well-known **Levenberg-Marquardt Technique** is applied to find the root of equation.





- Serval typical close path of figures on the car-body are designed with geometric specifications and its **real-time interpolation** is implemented directly in the level of CNC motion library.

Future work

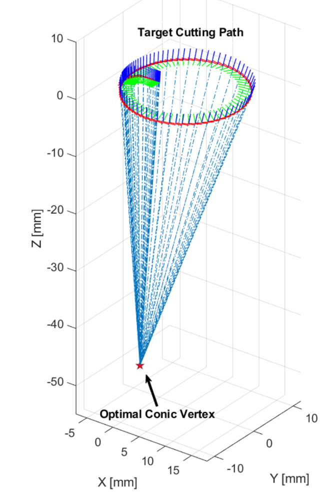
- The study will continue with tests on the redundant manipulator, including the simplification of the optimization model for the redundant axes, complying with the technological constraints, and the analysis of the solution of the optimal strategy (or local optimal solution).
- The convergence rate and strategy of the Damped Least Square (DLS) algorithm will need to be optimized and an attempt will be made to replace it with a more suitable optimization model, comparing the number of convergences and time complexity before and after.

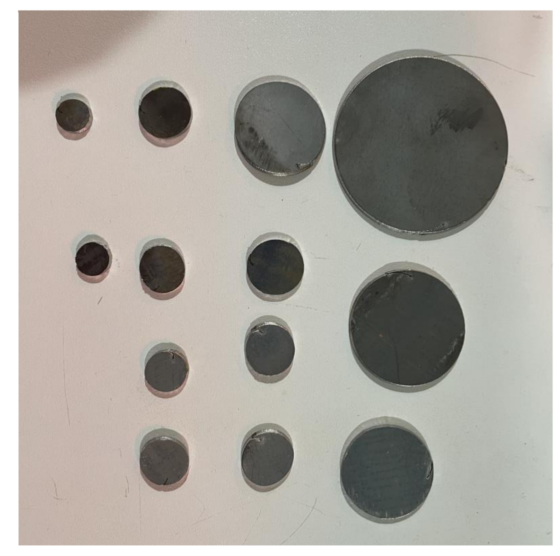
Submitted and published works

- Dual-point (TCP and virtual wrist center point) path planning for the manipulator optimal orientation management and singularity avoidance is to be studied for the next step, optimal time and energy saving benefits from the second virtual path planning from the dynamic aspect could be a further step of trajectory and control of the objective redundant manipulator.

Novel contributions

- An iterative numerical inverse kinematics is proposed for the wrist singularity avoidance with user-defined accuracy.
- Conic interpolation based on polar coordinate with the vertex.





Adopted methodologies

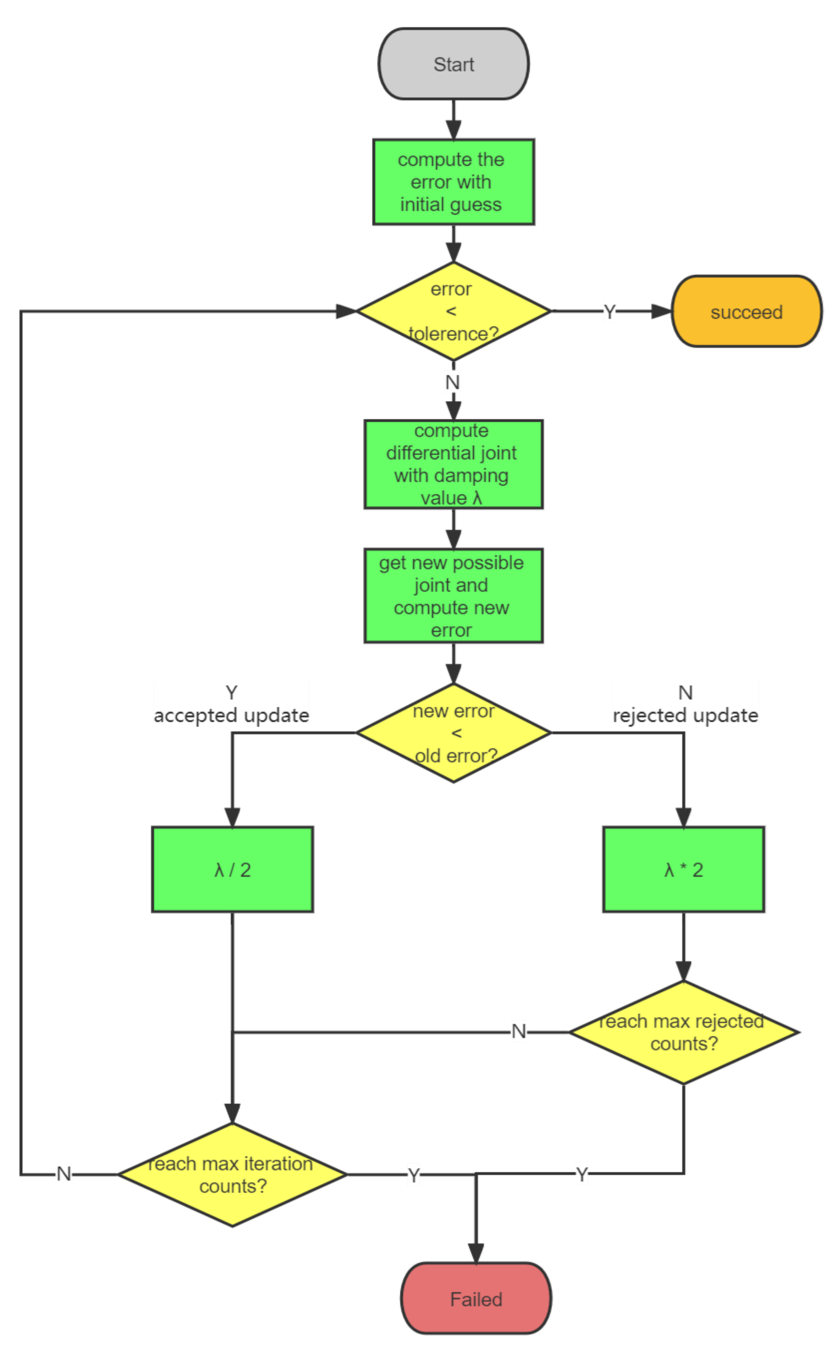
Damped Least Square (DLS)

The damped least square methods avoids many of pseudoinverse method's problems with singularity and can give a numerically stable method for selecting δq .

$$\min_{\delta q} \|\delta x - J(q)\delta q\|^2 + \lambda^2 \|\delta q\|^2$$

This can be rewritten according to singular value decomposition:

$$J = U\Sigma V^T = \sum_{i=1}^{m} \frac{\sigma_i}{{\sigma_i}^2 + \lambda^2} u_i {v_i}^T$$



Damped pseudoinverse:

$$J^{\dagger\lambda}(q) = J^T(q)(J(q)J^T(q) + \lambda^2 \mathrm{I})^{-1}$$

A weighting matrix is introduced in order to reflect the priority level of each constraints. The idea is to differently weigh the task space components when evaluating the tracking accuracy against the feasibility of joint motion.

$$\delta q = (J(q)WJ^T(q) + \lambda^2 We)^{-1} J^T(q)We$$

Where $\boldsymbol{W}$ is a diagonal matrix, and $\boldsymbol{e}$ is the reducing difference between the target and computed result after forward kinematics.

The adoption of $\boldsymbol{W}$ modifies the singular values and the singular vector of $\boldsymbol{J}$. On the other hand, an increased condition number implies a larger region in which the damping comes in effect, where an accurate and feasible solution might instead be possible .

List of attended classes

- 01NDLRV – Lingua italiana I livello(25/01/2022, CFU 3)
- 02SFURV – Programmazione scientifica avanzata in matlab (26/05/2022, CFU 6)
- 01RGBRV – Optimization methods for engineering problems(07/06/2022, CFU 6)
- 01RISRP – Public speaking (23/09/2022, CFU 1)

External activities:

- Numerical Optimization, IMT School for Advanced Studies Lucca, 2022