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XXXVII Cycle

Inverse Source Problem in Electromagnetics

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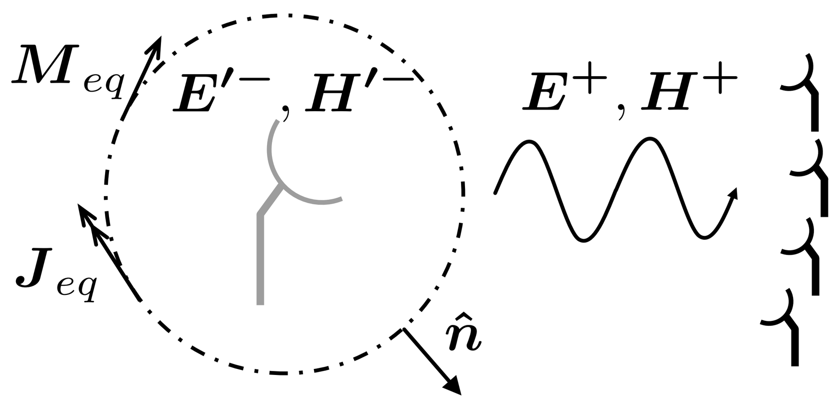
Research context and motivation

- Inverse source approaches in electromagnetics have shown their relevance for several applications in the past years, among which near-field to far-field transformations and **antenna diagnostics**. They leverage the **equivalent source theorem** which allow to transform a given radiation problem into an equivalent one through equivalent surface currents:



$$\boldsymbol{M}_{eq} = \hat{\boldsymbol{n}} \times \left(\boldsymbol{E}'^{-} - \boldsymbol{E}^{+}\right)$$
$$\boldsymbol{J}_{eq} = \hat{\boldsymbol{n}} \times \left(\boldsymbol{H}^{+} - \boldsymbol{H}'^{-}\right)$$

- The equivalent surface currents describe the radiation of the original source in an homogeneous medium through the **integral operators**

$$\mathcal{T}_{s,\boldsymbol{r}}\boldsymbol{f} = \hat{\boldsymbol{n}} \times \int_\Gamma \frac{e^{\mathrm{i}k|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\boldsymbol{f}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}' \qquad \mathcal{T}_{\boldsymbol{r}}\boldsymbol{f} = \mathrm{i}k\,\mathcal{T}_{s,\boldsymbol{r}}\boldsymbol{f} - \frac{1}{\mathrm{i}k}\mathcal{T}_{h,\boldsymbol{r}}\boldsymbol{f}$$

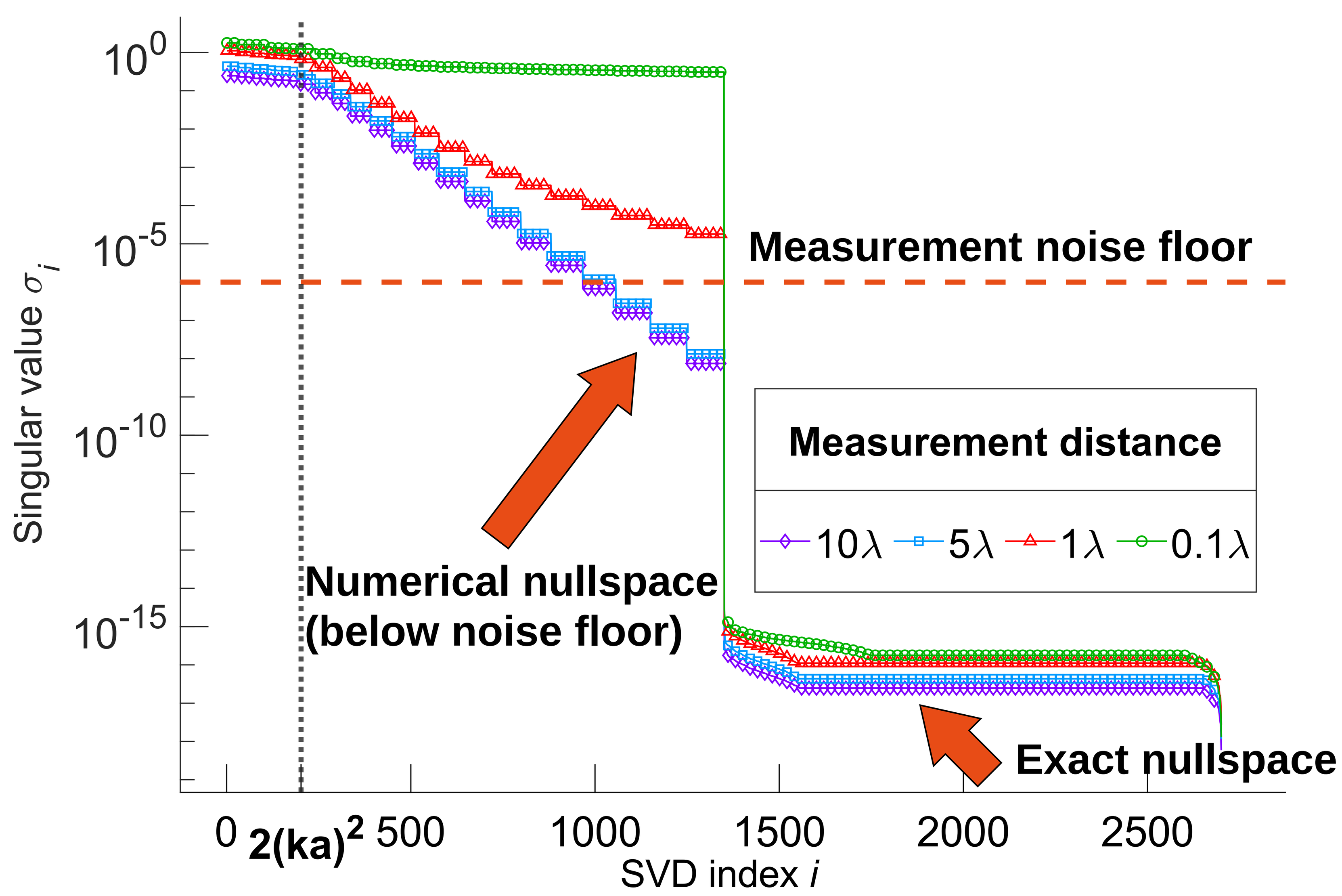
$$\mathcal{T}_{h,\boldsymbol{r}}\boldsymbol{f} = \hat{\boldsymbol{n}} \times \nabla \int_\Gamma \frac{e^{\mathrm{i}k|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\nabla_s \cdot \boldsymbol{f}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}' \qquad \mathcal{K}_{\boldsymbol{r}}\boldsymbol{f} = -\hat{\boldsymbol{n}} \times p.v.\int_\Gamma \nabla \times \frac{e^{\mathrm{i}k|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\boldsymbol{f}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}'$$

- The following linear system can be established:

$$\begin{bmatrix} -\mathcal{K}_{\boldsymbol{r}} & \mathcal{T}_{\boldsymbol{r}} \\ -\mathcal{T}_{\boldsymbol{r}} & -\mathcal{K}_{\boldsymbol{r}} \end{bmatrix} \begin{bmatrix} -\boldsymbol{M}_{eq} \\ \eta \boldsymbol{J}_{eq} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{n}} \times \boldsymbol{E}^{+} \\ \hat{\boldsymbol{n}} \times \eta \boldsymbol{H}^{+} \end{bmatrix}$$

Addressed research questions/problems

- The solution of the linear system above is not unique thus **pseudoinversion** must be used to get rid of the exact nullspace;
- Even if one valid solution is found the near-field description of the source is lost if far-field measurements are taken. This behavior is explained with the **spatial low-pass** filter action of the radiation operator which filters out the **evanescent modes**;



- The recovery of the evanescent modes from far-field measurements has not been addressed; a better evaluation of the near-field can lead to a better representation of the source under test and thus can have diagnostic value.

Submitted and published works

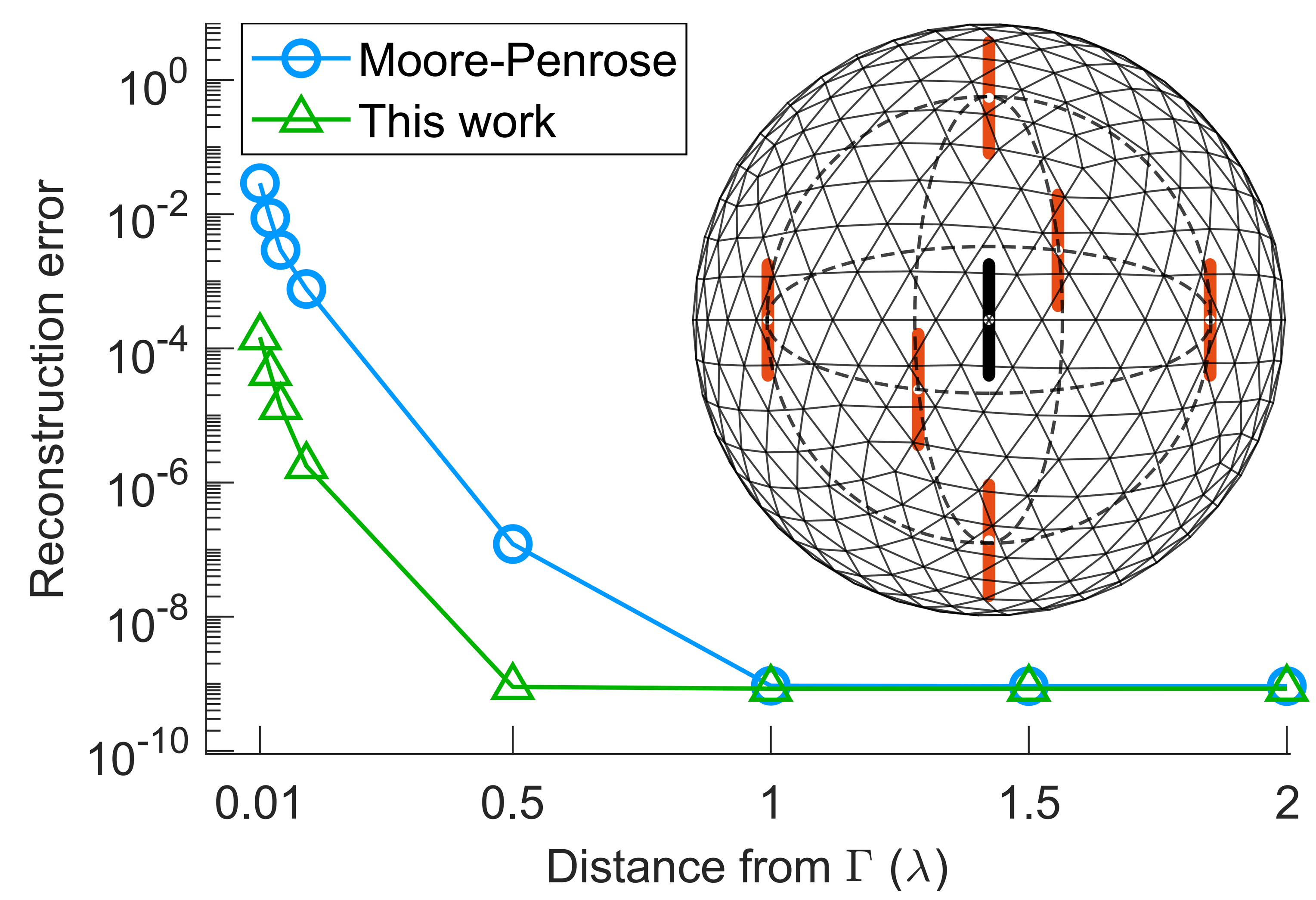
- E. Citraro, A. Dély, A. Merlini and F. P. Andriulli, "On a Constrained Pseudoinverse for the Electromagnetic Inverse Source Prolbem", IEEE AP-S/URSI, Denver, 2022
- P. Ricci, E. Citraro, A. Merlini, F. P. Andriulli, "Stabilized Single Current Inverse Source Formulations Based on Steklov-Poincaré Mappings", submitted to IEEE Antennas and Wireless Propagation Letters

Novel contributions

- Near-field information is injected into the system through *a priori* field vectors which populate the presented **constrained pseudoinverse**

$$\mathbf{R}^{\ddagger} = \mathbf{R}^{(1)}\left(\mathbf{I} - \tilde{\mathbf{B}}\mathbf{B}^{\dagger}\right) + \tilde{\mathbf{A}}\mathbf{B}^{\dagger}, \quad \mathbf{B}^{\dagger}\mathbf{B} = \mathbf{I}$$

- The evanescent fields are contained in $\tilde{\mathbf{A}}, \mathbf{B}, \tilde{\mathbf{B}}$ while the radiating far-fields are solved through $\mathbf{R}^{(1)}$;
- The scheme above leads to a better near-field content in the solution, and its reconstruction capability has been tested for an Hertzian dipole source:



Adopted methodologies

- The **Vector Spherical Harmonics** base has been used to gain a ground truth on the radiation operators before discretizing the problem with the local Rao-Wilton-Glisson basis function:

$$\mathcal{T}_{\boldsymbol{r}}(\boldsymbol{X}_{lm}) = -\frac{a}{r}\mathbb{J}_l(ka)\mathbb{H}_l^{(1)}(kr)\,\boldsymbol{U}_{lm}, \quad \mathcal{T}_{\boldsymbol{r}}(\boldsymbol{U}_{lm}) = \frac{a}{r}\mathbb{J}_l'(ka)\mathbb{H}_l^{(1)\prime}(kr)\,\boldsymbol{X}_{lm}$$
$$\mathcal{K}_{\boldsymbol{r}}(\boldsymbol{X}_{lm}) = \mathrm{i}\frac{a}{r}\mathbb{J}_l(ka)\mathbb{H}_l^{(1)\prime}(kr)\,\boldsymbol{X}_{lm}, \quad \mathcal{K}_{\boldsymbol{r}}(\boldsymbol{U}_{lm}) = -\mathrm{i}\frac{a}{r}\mathbb{J}_l'(ka)\mathbb{H}_l^{(1)}(kr)\,\boldsymbol{U}_{lm}$$

- The general pseudoinverse properties have been investigated

$$\mathbf{R}\mathbf{R}^{\dagger}\mathbf{R} = \mathbf{R} \quad \text{(i)} \qquad \mathbf{R}^{\dagger}\mathbf{R}\mathbf{R}^{\dagger} = \mathbf{R}^{\dagger} \quad \text{(ii)}$$
$$(\mathbf{R}\mathbf{R}^{\dagger})^{*} = \mathbf{R}\mathbf{R}^{\dagger} \quad \text{(iii)} \qquad (\mathbf{R}^{\dagger}\mathbf{R})^{*} = \mathbf{R}^{\dagger}\mathbf{R}. \quad \text{(iv)}$$

Future work

- Application of the constrained pseudoinverse to more complicated geometries and less symmetric radiation patterns;
- Further generalization of the pseudoinverse and exploration of the constrained spaces;
- Real case scenario: from measurements to synthetic reconstruction;
- Acceleration of the inversion through the preconditioned MLFMA.

List of attended classes

- 01DPJRV – Lens antennas: Fundamentals and present applications (7/12/2021, 13.33)
- 01UIZRV – Microwave sensing and imaging for innovative applications in health and food industry (22/3/2022, 33.33)
- 01DOBRV – Mathematical-physical theory of electromagnetism (20/6/2022, 20.00)
- 01UJDRV – Integral operators and fast solvers: a cross-disciplinary excursus on the best of FFT'companions (13/9/2022, 35)
- 02LWHRV – Communication (12/8/2022, 6.67)
- 08IXTRV – Project management (12/8/2022, 6.67)
- 01RISRV – Public speaking (5/9/2022, 6.67)