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XXXVII Cycle

Quantum-related approaches for solving optimization problems

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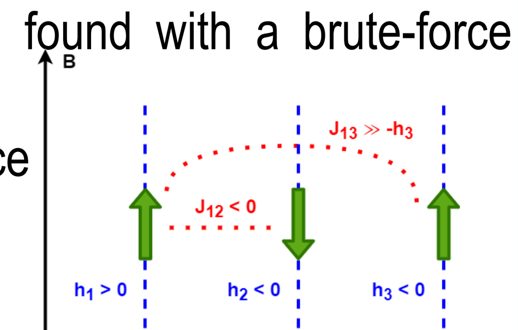
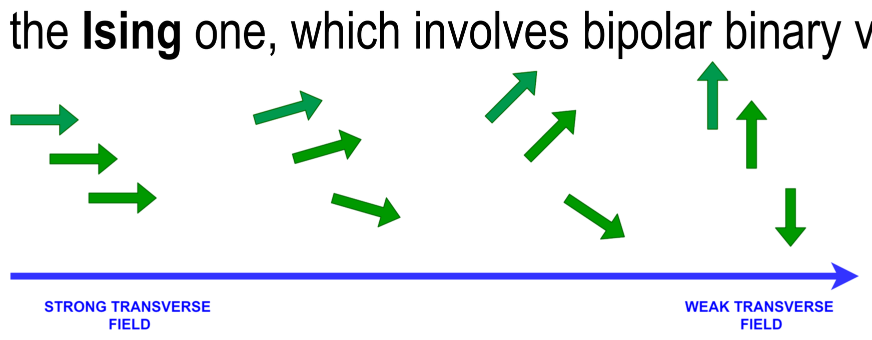
Research context and motivation

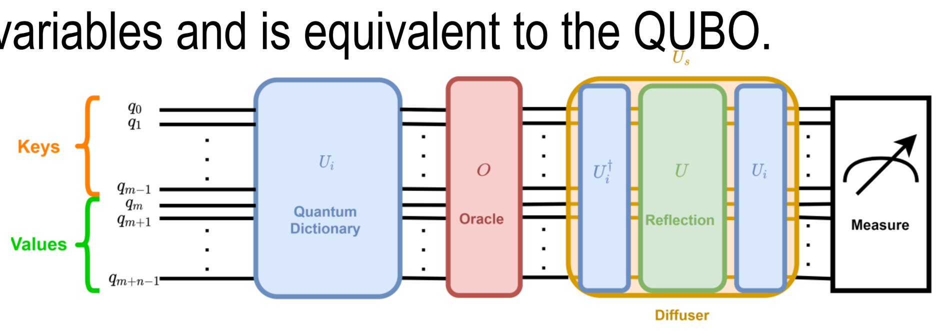
Combinatorial optimization (CO) problem goal is finding an input configuration which minimizes a cost function. The optimal solution can always be found with a brute-force approach, but the time required increases exponentially with the number of variables. Deterministic exploration of the solutions space also can be exploited. However, they are not suitable for some optimization problems, such as multimodal ones, and they can require a significant amount of time to achieve convergence.



Because of their limitations, heuristic approaches are commonly employed to find optimal or sub-optimal solutions to large-scale problems. Whereas many new classic approaches have been proposed in recent years, they are not always completely satisfactory in terms of execution time or accuracy. Therefore, the exploitation of quantum computers was proposed to obtain a speed-up by exploiting its intrinsic parallel computational capabilities.

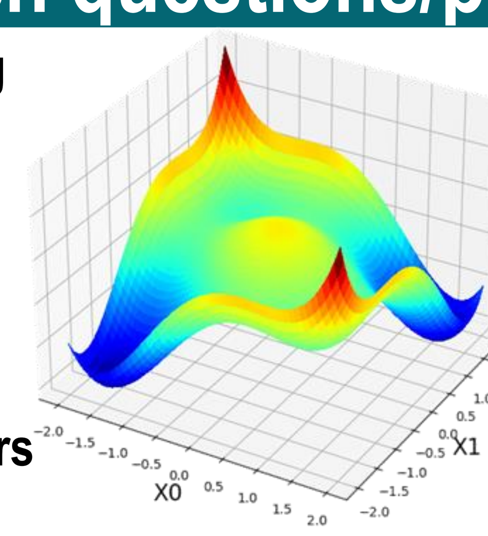
A **special-purpose quantum computer**, called **quantum annealer**, was theorized, and algorithms for **general-purpose quantum computers** were explored. The most feasible formulations for solving CO problems with quantum computers are the **Quadratic Unconstrained Binary Optimization** (**QUBO**) one, involving unipolar binary variables, and the **Ising** one, which involves bipolar binary variables and is equivalent to the QUBO.

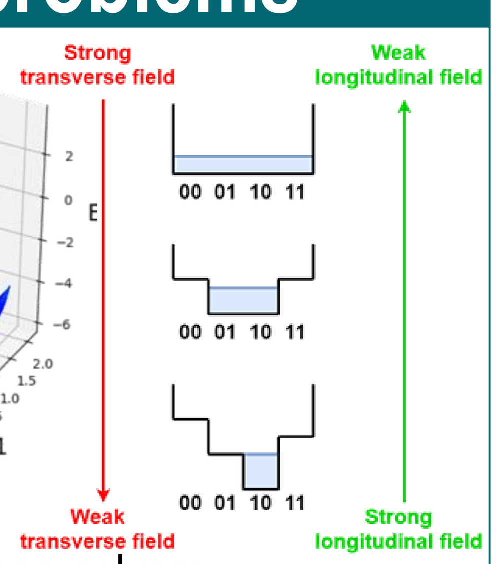




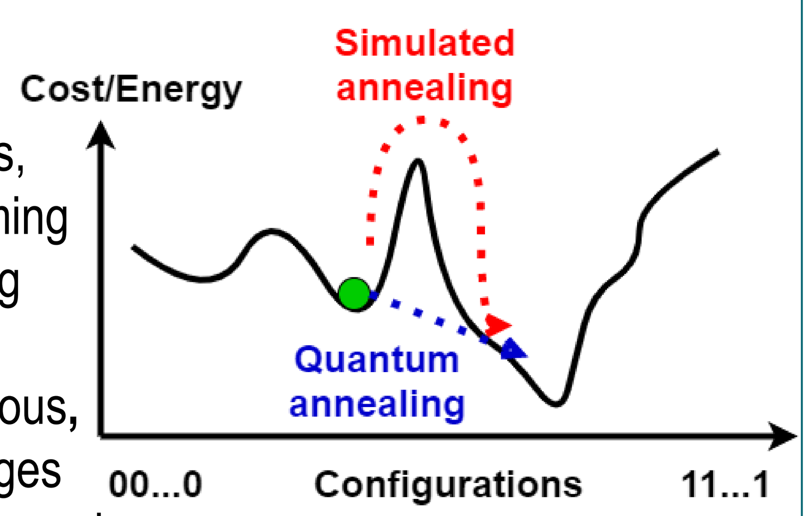
Addressed research questions/problems

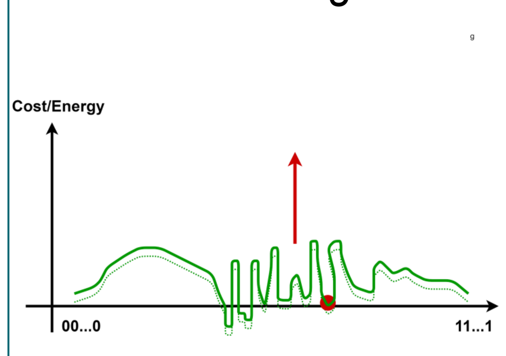
- Current quantum computers have strong limitations in terms of available qubits and functionality, due to not ideal phenomena, high costs and extreme environmental requirements. These limitations do not permit effectively to exploit quantum computing. **Emulators** of quantum computers can be useful for approximating ideal qubits performance and obtaining **on-premises** solvers.

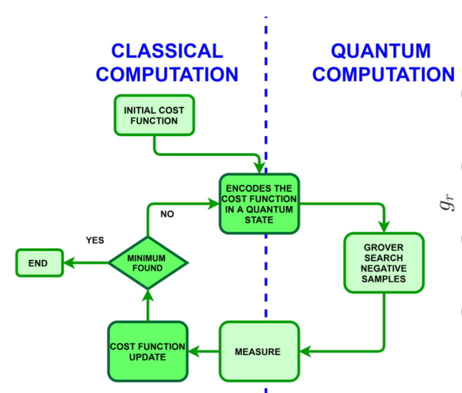


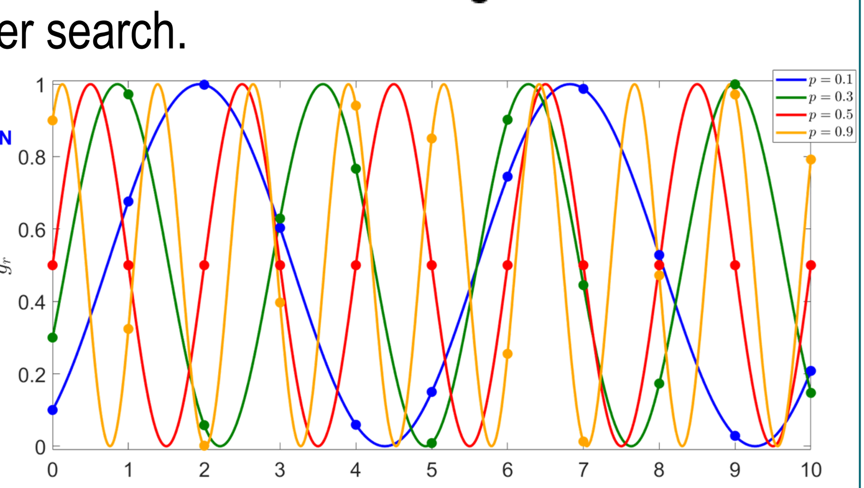


- Local search-based approaches, e.g. simulated annealing, are more effective in exploring wide and flat energy profiles, while quantum annealers, which can exploit the tunnelling effect for performing a global search, are more efficient for overcoming high and narrow barriers. Since the real-world problems energy profiles are usually heterogeneous, **hybrid solvers**, able to exploit both the advantages of local and global search, can perform a better search.

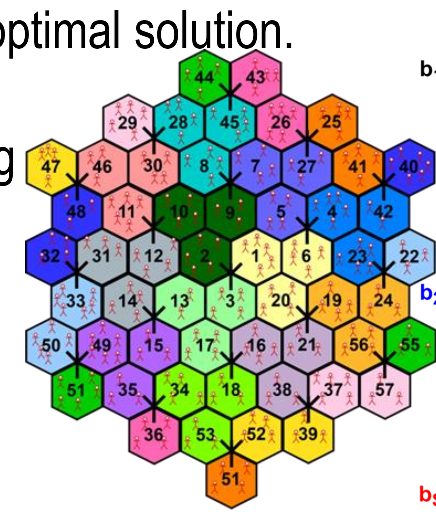


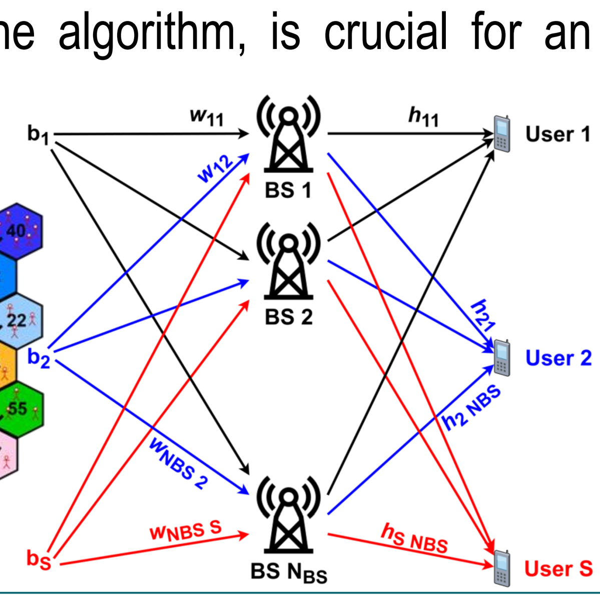






- A relevant quantum circuit model paradigm-based algorithm is the **Grover Adaptive Search** (**GAS**). Optimizing its degrees of freedom, i.e., the number of **Grover Search** (**GS**) iterations in each call and the condition to stop the algorithm, is crucial for an effective and fast convergence to the optimal solution.
- For exploiting quantum or quantum-based approaches for solving real-world problems, they should be written in a compatible formulation, such as **Ising** or **QUBO**.



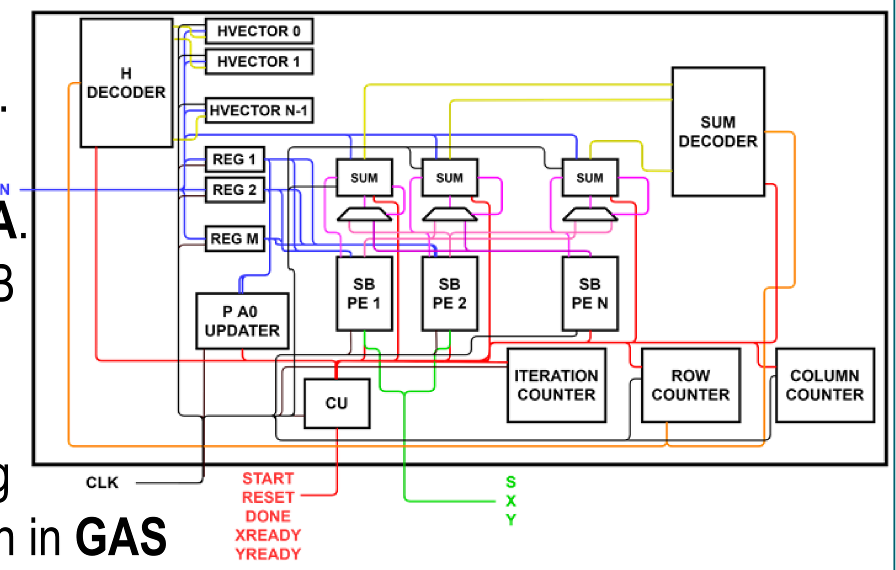


Submitted and published works

- L. Giuffrida, D. Volpe, G. A. Cirillo, M. Zamboni and G. Turvani, "Engineering Grover Adaptive Search: Exploring the Degrees of Freedom for Efficient QUBO Solving," in *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, 2022, doi: 10.1109/JETCAS.2022.3202566.

Novel contributions

1) Development of Python implementation of **Path Integral Quantum Monte Carlo** (**PI**) quantum annealing emulation algorithm for Ising problems.
2) Proposal and development of Python implementations of four **hybrid algorithms**, which combine PI and two optimization techniques of simulated annealing, i.e., **Parallel Tempering** and **Population Annealing**, for solving Ising problems.
3) Development of Python implementation of **Simulated Bifurcation** quantum-inspired algorithm for solving Ising problems, which supports **Adiabatic**, **Ergodic**, **Ballistic** and **Discrete** evolution of the non-linear Kerr oscillator network.
4) Development of a **fixed-point architecture** for Simulated Adiabatic Bifurcation in **VHDL**.
5) Development of a **GPU implementation** for Simulated Adiabatic Bifurcation in **CUDA**.
6) Development and optimization of a MATLAB implementation of the **Digital Annealing** algorithm for QUBO problems.
7) Proposal of a new mechanism for managing the number of rotations for each GS iteration in **GAS** algorithm and three **dynamic threshold-based** mechanisms for stopping it.
8) Improvement of an existing QUBO formulation for solving a **telecommunication-related problem**.

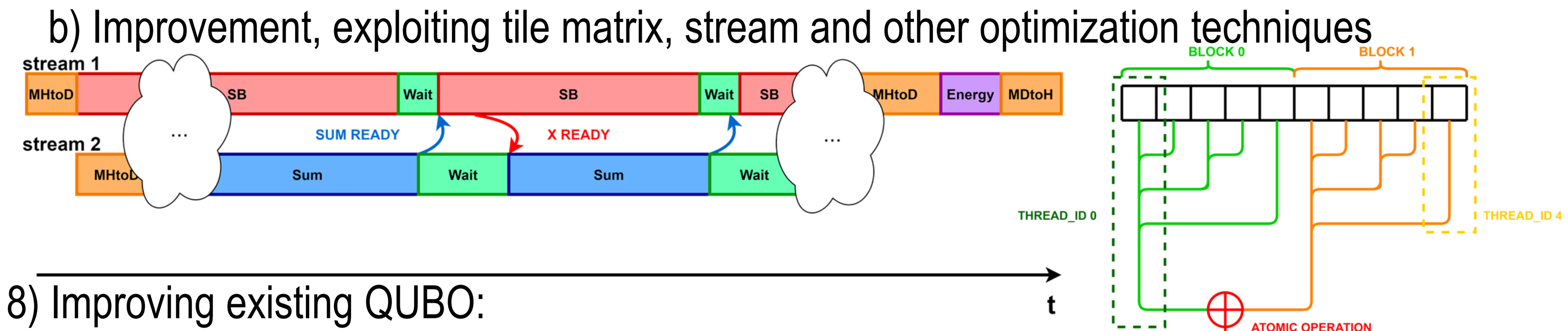


Adopted methodologies

1,2,3,6,7) Software implementation of quantum-related optimization algorithms:
   a) Study and analysis of the algorithm presented in the state-of-art, identifying the best parameters combination and comparing results of different solvers
   b) Improvement of the state-of-art algorithms or proposal of new approaches combining them to take the best of each studied approach or proposal of new approaches for managing algorithm degrees of freedom

4) FPGA architecture of the quantum-inspired algorithm:
   a) Exploit a software model for evaluating the best number representation
   b) Functional verification and evaluation of the performance

5) GPU implementation:
   a) Development of a basic version and performance evaluation
   b) Improvement, exploiting tile matrix, stream and other optimization techniques



8) Improving existing QUBO:
   a) Study of the original formulation, removal of unnecessary constraints and reduction of the number of variables

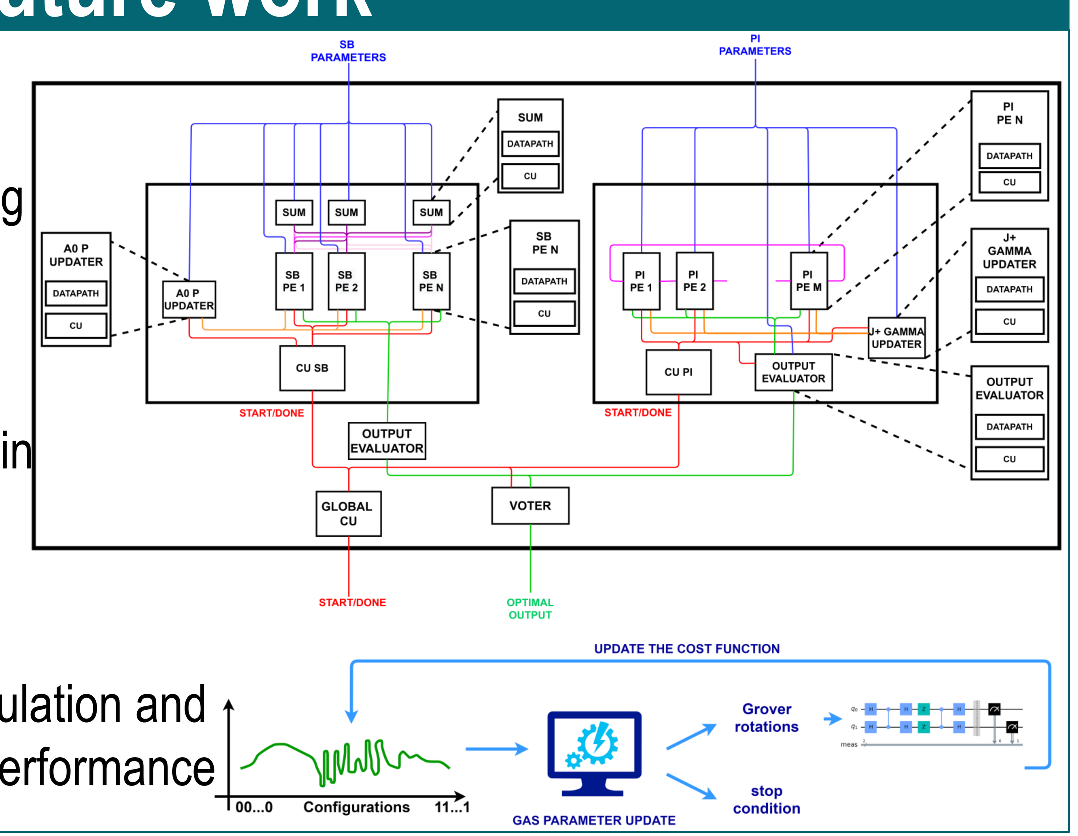
Future work

1,2,6)Development of FPGA and GPU implementations. Proposal of an architecture capable of supporting multiple approaches and hybrid versions.

3,4,5)Implementations improvement

7) Creation of an automatic toolchain for solving QUBO problems with quantum, quantum-inspired and quantum-assisted solvers

8) Further improvement of the formulation and comparison of different solvers performance



List of attended classes

- 01UNRRV – Entrepreneurship and start-up creation (5/07/2022, 8 cfu, 40 hours)
- 01URVOV – GPU programming (4/02/2022, 6 cfu, 60 hours)
- 01NOYOQ – Microelectronic systems (5/07/2022, 6 cfu, 60 hours)
- 01RGBRV – Optimization methods for engineering problems (7/06/2022, 6 cfu, 30 hours)
- 02SFURV – Programmazione scientifica avanzata in matlab (21/04/2022, 6 cfu, 30 hours)
- 01TAHIU – Quantum computing (12/07/2022, 4 cfu, 20 hours)
- 01DNHRV – System level low power techniques for IoT (15/07/2022, 4 cfu, 20 hours)

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