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XXXVII Cycle

Frequency-stabilized single current inverse source formulations

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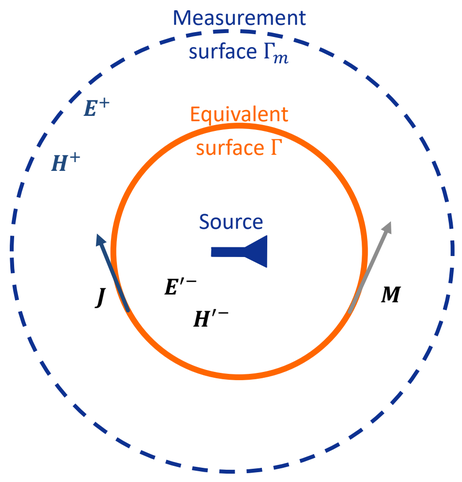
Research context and motivation

- Inverse source strategies are widely used for antenna diagnostics, NF-to-FF transformations, etc.
- The **inverse source problem** consists in the recovery of a configuration of sources radiating a given field and it relies on the **equivalence theorem**.
- The **Love condition** states that the fields inside the equivalent surface are null. It is often used for diagnostics purposes, as the equivalent sources that satisfy this condition are the **traces of the fields** on the surface.



Addressed research questions/problems

- The existing inversion strategies usually require **additional equations to enforce the Love condition**.
- **Single-current schemes** have smaller systems and are consequently faster, but they usually **do not enforce the Love condition**.
- The only single-current Love formulation in the literature relies on an **approximation of the Steklov-Poincaré operator**.
- The existing formulations suffer from **low-frequency breakdown**.

Novel contributions

- **Two single-current formulations** that enforce the Love condition without increasing the size of the matrix.

$$\left(-\mathcal{K}_{\mathbf{r}} - \mathcal{T}_{\mathbf{r}} \left(\tfrac{\mathcal{I}}{2} + \mathcal{K}\right)^{-1} \mathcal{T}\right) (-\boldsymbol{M_L}) = \hat{\boldsymbol{n}}_{\boldsymbol{r}} \times \boldsymbol{E}^+$$

$$\left(\mathcal{T}_{\mathbf{r}} + \mathcal{K}_{\mathbf{r}} \mathcal{T}^{-1} \left(\tfrac{\mathcal{I}}{2} + \mathcal{K}\right)\right) (\eta \boldsymbol{J_L}) = \hat{\boldsymbol{n}}_{\boldsymbol{r}} \times \boldsymbol{E}^+$$

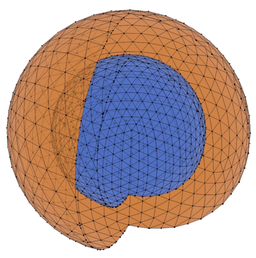
- A **stabilization in frequency of the novel formulations** obtained by discretizing the equivalent surface and the measurement surface and by leveraging the **quasi-Helmoltz projectors**.

$$\mathbb{P}_k \left(-\mathbf{K}_m - \mathbb{T}_m \left(\mathbb{G}/2 + \mathbb{K}\right)^{-1} \mathbf{T}\right) \mathbf{P}_k \mathbf{x} = \mathbb{P}_k \mathbf{e}_m$$

$$\mathbf{P}_k \left(\mathbf{T}_m + \mathbb{K}_m \mathbb{T}^{-1} \left(-\mathbb{G}^T/2 + \mathbf{K}\right)\right) \mathbf{P}_k \mathbf{y} = \mathbf{P}_k \mathbf{e}_m$$

- A stabilization in frequency of the **Steklov-Poincaré operator**.

$$\mathbb{P}_k^{-1} \left(\mathbb{G}/2 + \mathbb{K}\right)^{-1} \mathbf{T} \mathbf{P}_k$$

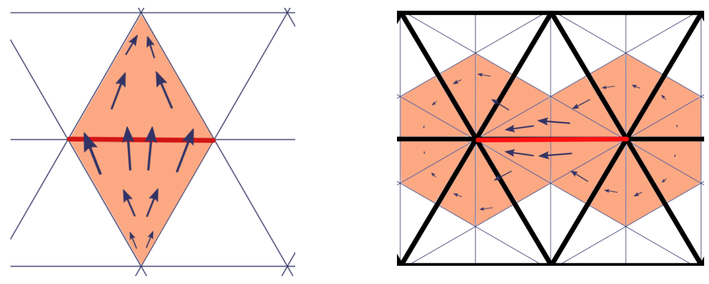


Submitted and published works

- Paolo Ricci, Adrien Merlini, Francesco P. Andriulli, "*On a Frequency-Stabilized Single Current Inverse Source Formulation*", IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting, Denver, U.S.A., 2022
- Carlo Baronio, Giulio Cosentino, Paolo Ricci, Clément Henry, Maxime Y. Monin, Adrien Merlini, Francesco P. Andriulli, "*On a Fast Solution Strategy for a Surface-Wire Integral Formulation of the Anisotropic Forward Problem in Electroencephalography*", IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting, Denver, U.S.A., 2022
- Arturo Micheli, Davide Consoli, Adrien Merlini, Paolo Ricci, Francesco P. Andriulli, "*Brain-Computer Interfaces: Investigating the Transition from Visually Evoked to Purely Imagined Steady-State Potentials*", International Engineering in Medicine and Biology Conference, Glasgow, United Kingdom, 2022
- Paolo Ricci, Ermanno Citraro, Adrien Merlini, Francesco P. Andriulli, "*Stabilized Single Current Inverse Source Formulations Based on Steklov-Poincaré Mappings*", submitted to IEEE Antennas and Wireless Propagation Letters

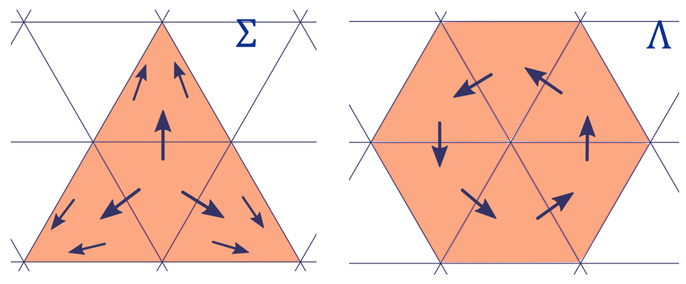
Adopted methodologies

- Both the **Rao-Wilton-Glisson** (RWG) and the **Buffa-Christiansen** (BC) **basis functions** are used.



- The quasi-Helmoltz projectors are obtained from the **Star-to-RWG** and the **Loop-to-RWG** transformation matrices.

$$\mathbf{P}_k = \mathbf{P}^{\Lambda H} k^{-1/2} + \mathrm{i}\mathbf{P}^{\Sigma} k^{1/2}$$
$$\mathbb{P}_k = \mathbb{P}^{\Sigma H} k^{-1/2} + \mathrm{i}\mathbb{P}^{\Lambda} k^{1/2}$$
$$\mathbf{P}^{\Sigma} = \mathbf{\Sigma}(\mathbf{\Sigma}^T \mathbf{\Sigma})^{+} \mathbf{\Sigma}^T$$
$$\mathbb{P}^{\Lambda} = \mathbf{\Lambda}(\mathbf{\Lambda}^T \mathbf{\Lambda})^{+} \mathbf{\Lambda}^T$$
$$\mathbf{P}^{\Lambda H} = \mathbf{I} - \mathbf{P}^{\Sigma}$$
$$\mathbb{P}^{\Sigma H} = \mathbf{I} - \mathbb{P}^{\Lambda}$$



Future work

- Extend the single-current formulations to deal with interior resonances.
- Investigate possible industrial applications.
- Study whether and how machine learning tools can be exploited in this context.

List of attended classes

- 01DPJRV – Lens antennas: Fundamentals and present applications. (didattica di eccellenza) (7/12/2021, 10 hours)
- 01UNYRV – Personal branding (17/1/2022, 1 hour)
- 01UNXRV – Thinking out of the box (17/1/2022, 1 hour)
- 01UNVRV – Navigating the hiring process: CV, tests, interview (18/1/2022, 2 hours)
- 08IXTRV – Project management (19/1/2022, 5 hours)
- 01SWPRV – Time management (19/1/2022, 2 hours)
- 01RISRV – Public speaking (20/1/2022, 5 hours)
- 02RHORV – The new Internet Society: entering the black-box of digital innovations (25/1/2022, 6 hours)
- 01SYBRV – Research integrity (26/1/2022, 5 hours)
- 02LWHRV – Communication (27/1/2022, 5 hours)
- 01SHMRV – Entrepreneurial Finance (27/1/2022, 5 hours)
- 01SWQRV – Responsible research and innovation, the impact on social challenges (30/1/2022, 5 hours)
- 01UIZRV – Microwave sensing and imaging for innovative applications in health and food industry (22/3/2022, 20 hours)
- Advancing Your Scientiic Presentation (Nature Masterclasses) (27/4/2022, 4 hours)
- Scientific Writing and Publishing (Nature Masterclasses) (27/4/2022, 6 hours)
- Narrative Tools for Researchers (Nature Masterclasses) (27/4/2022, 2 hours)
- Summer School on Boundary Element Methods (Graz University of Technology) (28/6/2022, 24 hours)
- 01DOBRV – Mathematical-physical theory of electromagnetism (11/7/2022, 15 hours)
- 01UJDRV – Integral operators and fast solvers: a cross-disciplinary excursus on the best of FFT'companions (13/9/2022, 21 hours)

Acknowledgement

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