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XXXVII Cycle

Calderón Preconditioners for the TD-EFIE discretized with Convolution Quadratures

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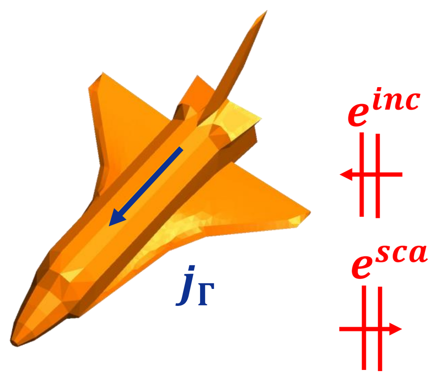
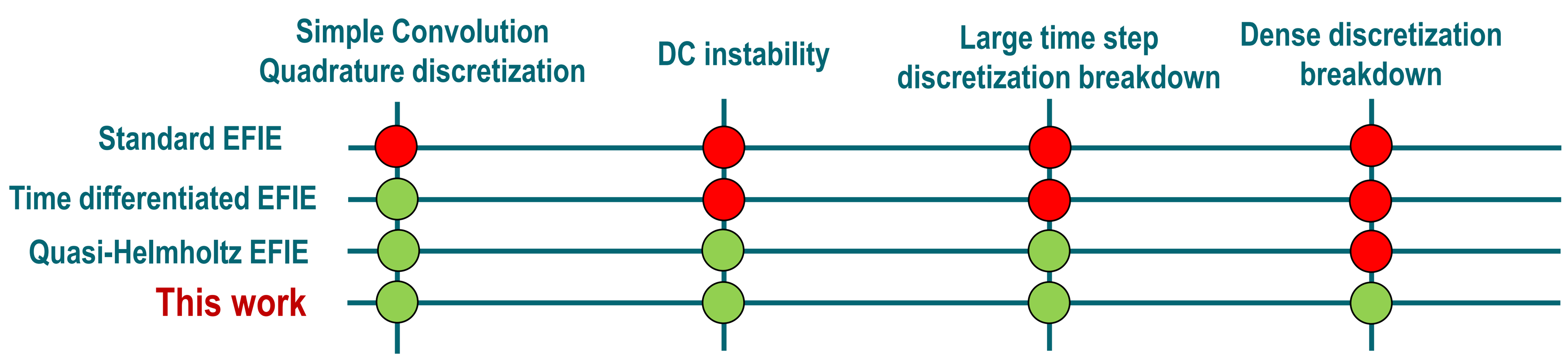
Research context and motivation

- *In the context* of **electromagnetic simulation**, integral equations are widely spread to evaluate fields for scattering and radiation from arbitrarily shaped. It consists of simulating the induced current j_Γ illuminated by an incident electromagnetic field, on the scatterer surface. The current thus evaluated radiates the scattered field. In the case of perfect electrically conducting objects, the time domain electromagnetic field integral equation (TD-EFIE) is mainly used,



$$\eta_0 \mathcal{T} j_\Gamma(\boldsymbol{r},t) = -\hat{\boldsymbol{n}} \times \boldsymbol{e}^{inc}(\boldsymbol{r},t)$$

- *Motivation*: All known time-domain integral equation techniques for simulating scattering and radiation from arbitrarily shaped, perfect electrically conducting objects, suffer from one or more of the following shortcomings

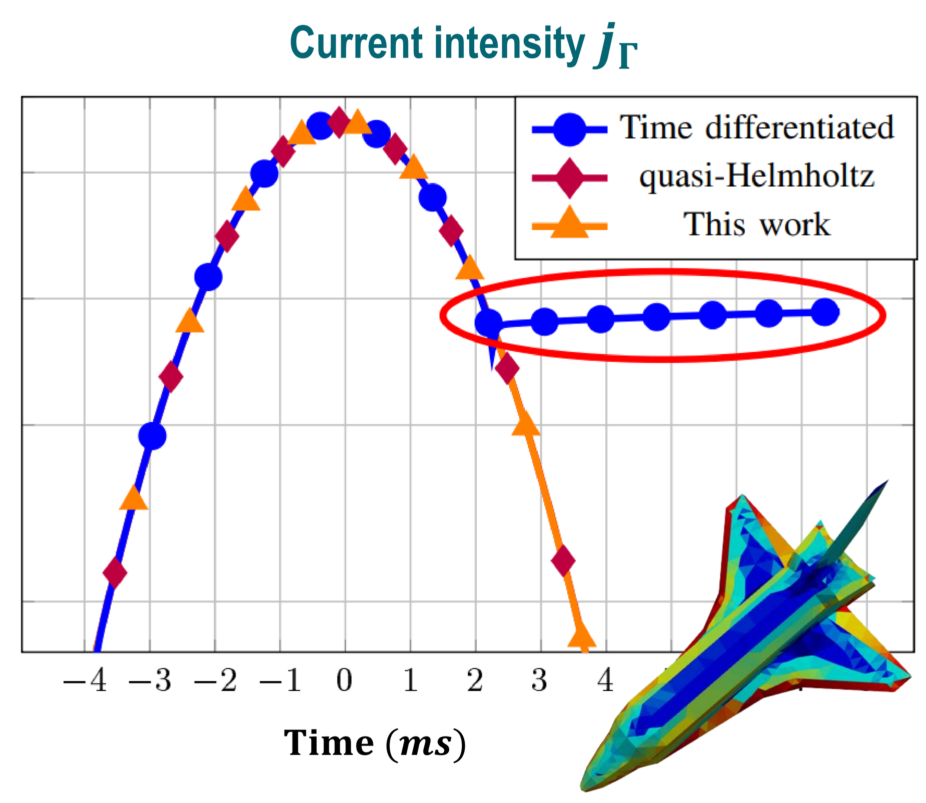


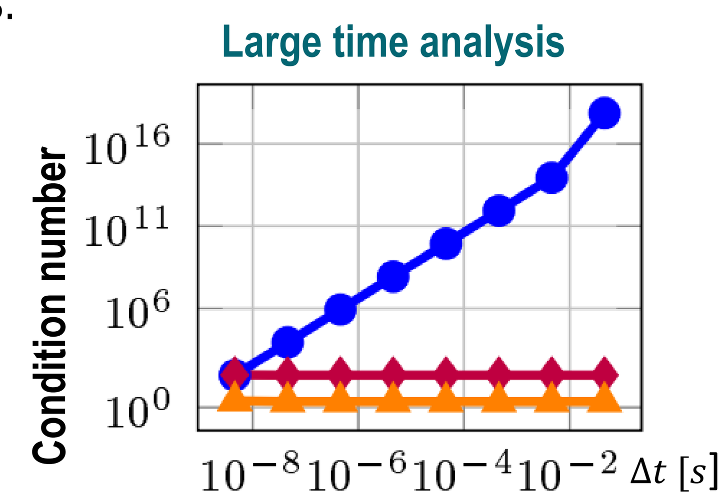
Novel contributions

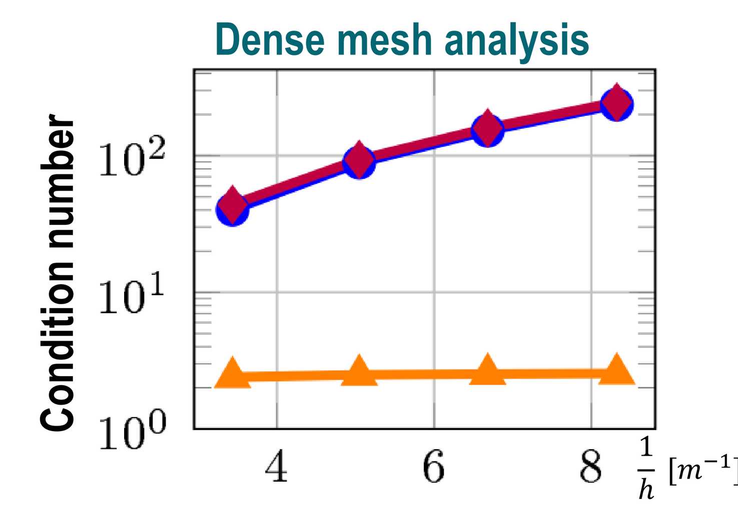
- The new proposed formulation leads to the following **Marching-On-In-Time**

$$\eta_0[\mathbf{T}^2]_0 \mathsf{J}_i \sum_{j=0}^{i} -c_0{}^{-1}[\dot{\mathbb{T}}_s]_j \mathbf{G}_m^{-1} \mathsf{E}_{i-j} + c_0[\mathbb{T}_h]_j \mathbf{G}_m^{-1} \mathsf{E}_{i-j}^{INT} - \eta_0 \sum_{j=1}^{i} [\mathbf{T}^2]_j \mathsf{J}_{i\_j}$$

- At the cost of adding some matrix-vector multiplications, this MOT is **stable** and **well-conditioned** as suggested by the following results:

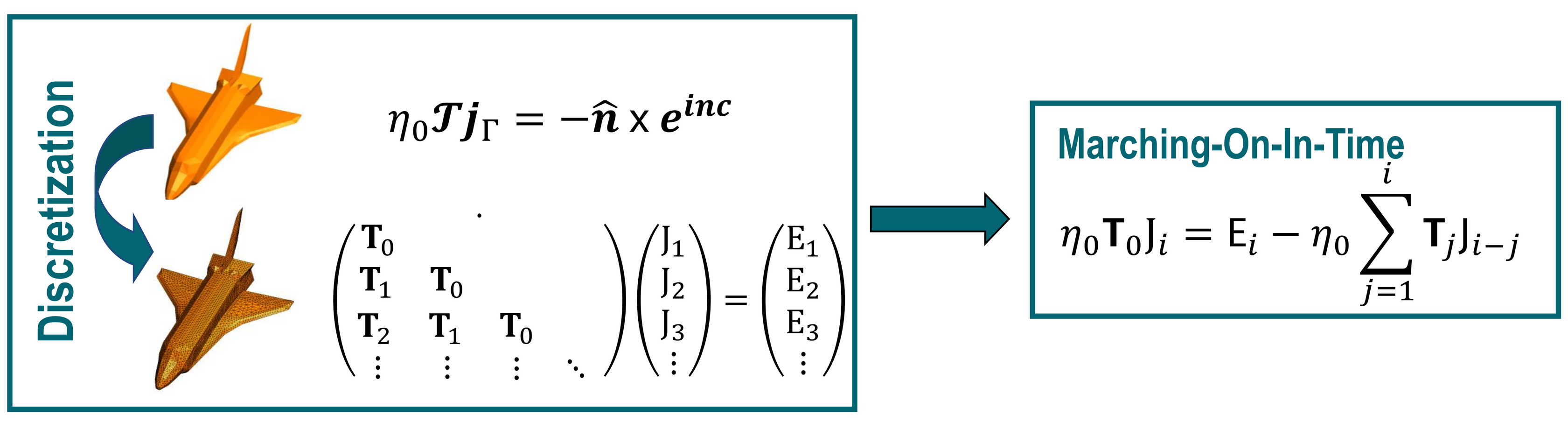






Addressed research questions/problems

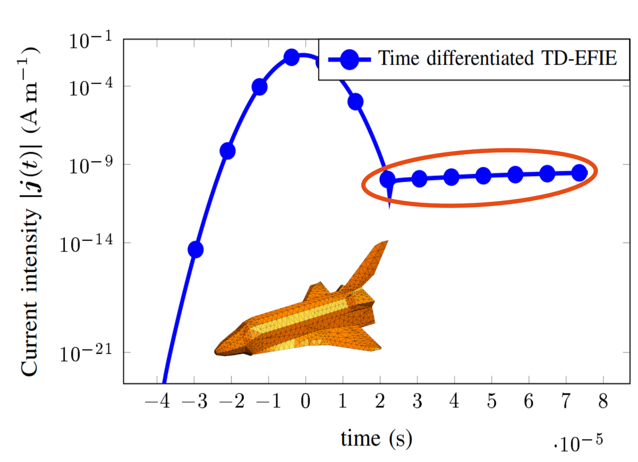
- The TD-EFIE formulation is discretized in space and time via the **Method of Moment** combined with the **convolution quadrature method** leading to a large Time/Space matrix system. Then, it is recast in a Marching-On-In-time (MOT) scheme.



- *The first limitation* of this discretization is a **time integration** on the TD-EFIE operator which is plugging the direct discretization with the Convolution quadratures. In the literature, charge accumulation methods or time differentiated formulations overcome this by removing the time integration from the discretized operators.

$$\mathcal{T}(\boldsymbol{f})(\boldsymbol{r},t) = \frac{-1}{c_0}\frac{\partial}{\partial t}\mathcal{T}_s(\boldsymbol{f})(\boldsymbol{r},t) + c_0 \int_{-\infty}^{t} \mathcal{T}_h(\boldsymbol{f})(\boldsymbol{r},t')dt'$$

- *The second limitation* is the **direct current instability** (DC-instability). This instability manifests at late time simulation by the presence of a spurious linear current leaving in the null space of the operators. To remove them, a complete time domain preconditioning must be done on the continuous operator.



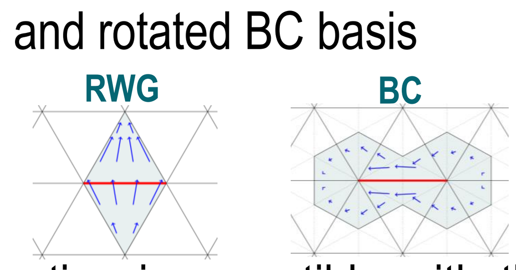
- *The last limitation* is the **conditioning of the MOT**. Indeed, to accelerate the computation, iterative solvers are required. However, their competitiveness is grandly reduced with ill-conditioned MOT. The current formulation suffers from an ill-conditioning at large time step and dense mesh.

Adopted methodologies

- The Calderón preconditioning is based on the **Calderón identity**, where $\mathcal{I}$ is the identity operator and $\mathcal{K}$ is a compact operator.

$$\mathcal{T}^2 = \frac{\mathcal{I}}{4} - \mathcal{K}^2$$

- A **mixed discretization** using RWG basis function as source and rotated BC basis function (its dual basis) leads to a well-conditioned MOT at large time step and dense mesh.



- The naive discretization of this operator implies time integration incompatible with the convolution quadratures. However, the **time integrations are removed** by using commutative and cancellation properties.

$$\mathcal{T}^2 j_\Gamma(\boldsymbol{r},t) = c_0^{-2}\frac{\partial^2}{\partial t^2}\mathcal{T}_s^2 j_\Gamma(\boldsymbol{r},t) - \mathcal{T}_s\mathcal{T}_h j_\Gamma(\boldsymbol{r},t) - \mathcal{T}_h\mathcal{T}_s j_\Gamma(\boldsymbol{r},t)$$

- Whereas $\mathcal{T}$ has the null space, $\mathcal{T}^2$ **has no null space**. This has been shown and named the dottrick demonstration.

$$\mathcal{T}^2 j_\Gamma(\boldsymbol{r},t) = \mathbf{0} \Rightarrow j_\Gamma(\boldsymbol{r},t) = \mathbf{0}$$

Future work

- The combined field integral equation (TD-CFIE) is the only formulation free from resonances. Future work will be to extend these results to the CFIE to have non-resonances, free from DC-instability and well-conditioned formulation.
- Fast technics such as the Adaptive Cross Section (ACA) or the plane wave time domain (PWTD) algorithm must be coupled with the formulation to have a competitive solver.

Acknowledgement

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MARIE CURIE ACTIONS

Submitted and published works

- Cordel P., Dély A., Merlini A., Andriulli P. F., Calderón Preconditioners for the TD-EFIE discretized with the Convolution Quadratures, IEEE AP-S/URSI 2022, Denver, Colorado, USA

List of attended classes

- 02LWHRV – Communication (30/8/2022, 6.67)
- 02RBYKI – From science to business, how to technology out of laboratories and into practical applications (8/7/2022, 26.67)
- 01UDPJRV – Lens antennas, Fundamentals and present applications (7/12/2021, 13.33)
- 01DOBRV – Mathematical-physical theory of electromagnetism (20/6/2022, 20)
- 01SFVRV– Metamaterial: Theory and Multiphysics applications (1/4/2022, 33.33)
- 01UIZRV – Microwave sensing and imaging for innovative applications in health and food industry (22/3/2022, 33.33)
- 01RISSRV– Public speaking (5/9/2022, 6.67)
- 01QORRV – Thinking out of the box (7/2/2022, 1.33)

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