ScuDo
Scuola di Dottorato – Doctoral School
WHAT YOU ARE, TAKES YOU FAR

XXXVII Cycle

New paradigms in low-frequency modelling for EMC industrial application

Johann Bourhis

Supervisor: Prof. Francesco P. Andriulli

Research context and motivation

- Several application scenarios of high industrial impact, including the modelling and design of high-speed circuitry and moderate to low frequency devices is a fundamentally important challenge in advanced electromagnetics industry.

- In fact, if the frequency of operation in these scenarios is too high to leverage on quasi-static solvers, it is still low enough for breaking the majority of available simulation packages. The problem becomes even more challenging in the presence of spatially multiscale geometries where regions of very fine details cohabit with large surfaces and reflectors.

- This project will tackle this problem by proposing a new family of low-frequency solvers which will be able to continuously enable predictions from extremely low frequency scenarios till the middle frequency range.

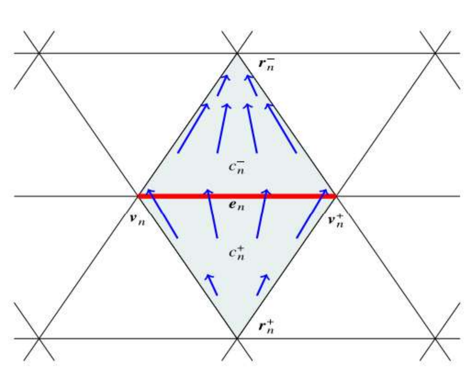
Addressed research questions/problems

- The electric field integral equation (EFIE) is widely used in computational EM for simulating radiation problems with perfectly electrically conducting (PEC) objects.

→ **This formulation suffers from ill-conditioning issues at low-frequency.**

$$\mathrm{i}k\int_{\Gamma\times\Gamma} G(\mathbf{x},\mathbf{y})\;\mathbf{J}(\mathbf{y})\cdot\mathbf{J}'(\mathbf{x})\;\mathrm{d}S(\mathbf{x})\mathrm{d}S(\mathbf{y}) + \frac{1}{\mathrm{i}k}\int_{\Gamma\times\Gamma} G(\mathbf{x},\mathbf{y})\;\nabla\cdot\mathbf{J}(\mathbf{y})\nabla\cdot\mathbf{J}'(\mathbf{x})\;\mathrm{d}S(\mathbf{x})\mathrm{d}S(\mathbf{y})$$

$$= -\frac{1}{\eta}\int_{\Gamma}\mathbf{E}^{inc}(\mathrm{x})\cdot\mathbf{J}'(\mathrm{x})\;dS(\mathrm{x})$$

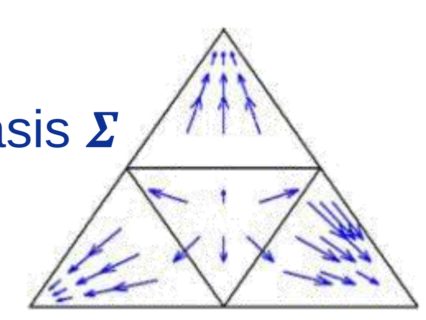


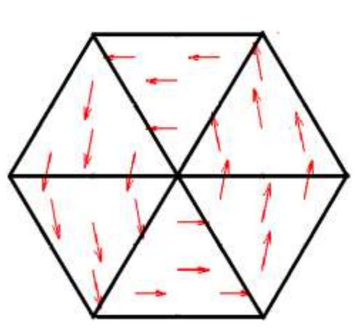
Discretization with RWGs

$$\boldsymbol{T} = \mathrm{i}k\boldsymbol{T}_s + \frac{1}{\mathrm{i}k}\boldsymbol{T}_h$$

- The loops and stars decomposition consists in splitting the current into solenoidal and non-solenoidal parts that allows to get rid of the ill-conditioning by rescaling the equation with respect to those two contributions.

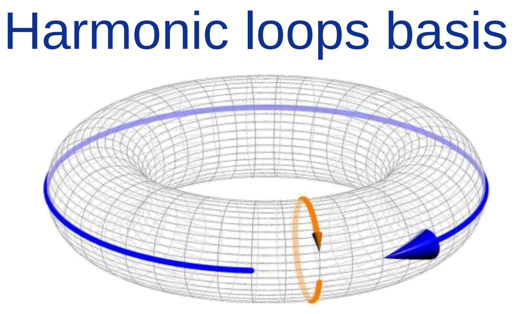
Star basis $\boldsymbol{\Sigma}$





Local loop basis $\boldsymbol{\Lambda}$

Harmonic loops basis $\boldsymbol{H}$



→ **The identification of the global loops may be a challenging step of the method, even in the absence of junctions.**

- The quasi-Helmholtz projectors avoid to identify the loops by computing a projector over the non-solenoidal subspace. In addition, its orthogonal projector gives the projection over the solenoidal subspace. The projectors are used to build a well-conditioned system.

$$[\boldsymbol{\Sigma}]_{ij} = \begin{cases} 1 & \text{if the cell } j \text{ equals } c_i^+ \\ -1 & \text{if the cell } j \text{ equals } c_i^- \\ 0 & \text{otherwise} \end{cases} \qquad \begin{aligned} \boldsymbol{P}_{\Sigma} &= \boldsymbol{\Sigma}(\boldsymbol{\Sigma}^T\boldsymbol{\Sigma})^+\boldsymbol{\Sigma}^T \\ \boldsymbol{P}_{\Lambda H} &= \boldsymbol{I} - \boldsymbol{P}_{\Sigma} \end{aligned}$$

$$\underbrace{\boldsymbol{P}\left(\mathrm{i}k\boldsymbol{T}_s + \frac{1}{\mathrm{i}k}\boldsymbol{T}_h\right)\boldsymbol{P}}_{\mathrm{i}(\boldsymbol{P}_{\Lambda H}\boldsymbol{T}_s\boldsymbol{P}_{\Lambda H} + \boldsymbol{T}_h) + O(k)}\mathbf{y} = \boldsymbol{P}\mathbf{e} \qquad \text{with } \boldsymbol{P}\mathbf{y} = \mathbf{j}$$

- This method has been developed and tested on closed structures.

→ **How to extend it to more complex geometries?**

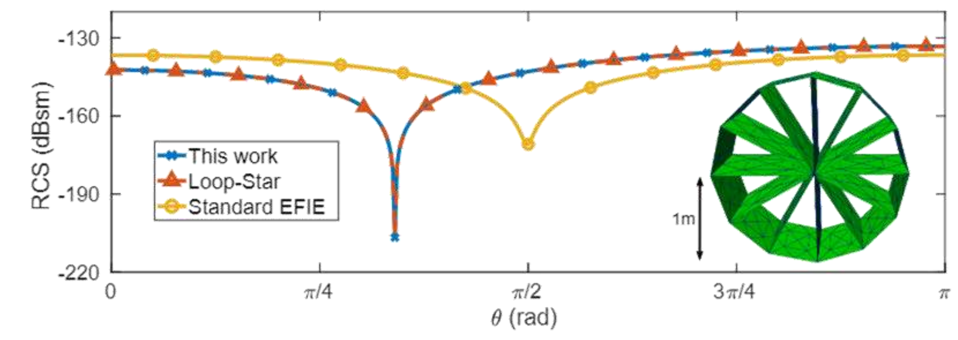
Submitted and published works

- Johann Bourhis, Adrien Merlini, Francesco P. Andriulli, "*Low-Frequency preconditioning using the Quasi-Helmholtz projectors for general problems with junctions.*", IEEE APS/URSI, Denver, 2022

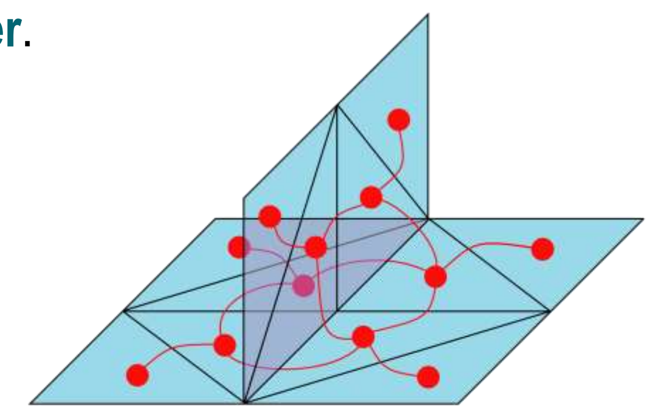
Novel contributions

- Extension of the use of the quasi-Helmholtz projectors for **open structures that contain junctions**. This study shows that the method can be applied to all kind of structures: closed object as well as open objects with junctions.

- Numerical experiments concerning the **scalability of algebraic multigrid solvers**, for inverting $\boldsymbol{\Sigma}^T\boldsymbol{\Sigma}$, shows that the simulations can be achieved in a quasi-linear number of operations in order to use the method in a **fast solver**.



Radar Cross Section obtained with and without preconditioning with a frequency of $1\cdot 10^4$ Hz (plane wave excitation).

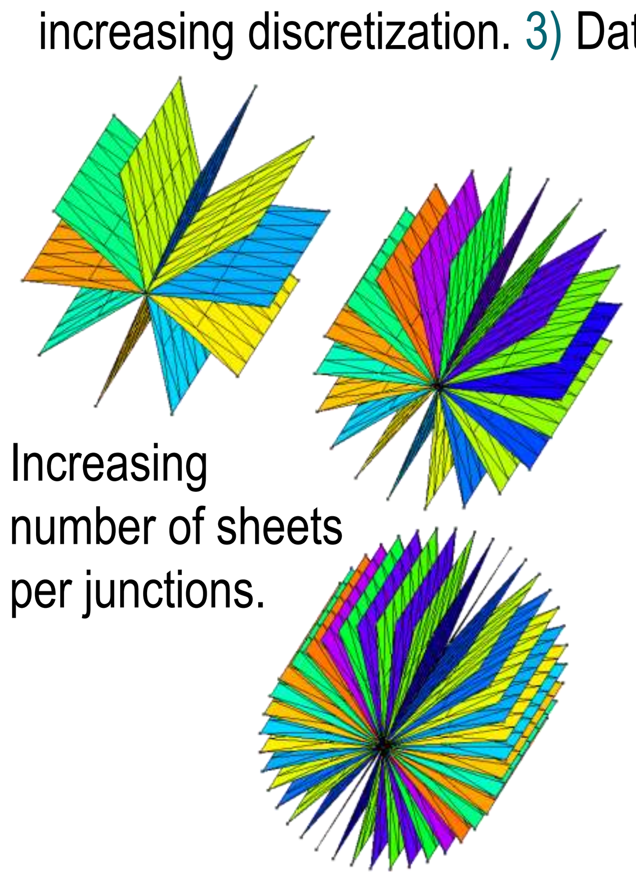


Additional nodal loops appear around the vertices of a junction, connecting the sheets pair by pair.

Adopted methodologies

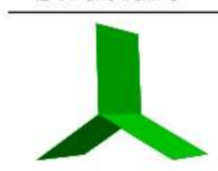
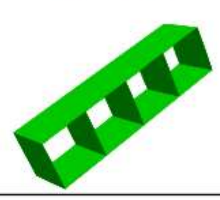
- First numerical experiments were done to observe the applicability of the quasi-Helmholtz projectors to complex structures: 1) Use of the software gmsh for constituting a bench of structures with particular interest (variable number of junctions). 2) Implementation in the laboratory's software *Atreyu* to adapt the computation to those cases. 3) Collect of the data showing the efficiency of the method for those new problems.

- Then, we focused on the scalability of this method with respect to an increasing number of sheets per junction and to the refinement of the mesh: 1) Implementation of an in-house algebraic multigrid solver. 2) Benchmark with an increasing number of junctions and an increasing discretization. 3) Data analysis and interpretation.



Increasing number of sheets per junctions.

Number of iterations achieved by AGMG for increasing discretization with a residual error below $1\cdot 10^{-8}$.

Structure	Unknowns	Iter.	Structure	Unknowns	Iter.
	$7.2\cdot 10^3$	21		$2.9\cdot 10^3$	18
	$6.9\cdot 10^4$	22		$2.8\cdot 10^5$	24
	$7.0\cdot 10^5$	22		$2.7\cdot 10^6$	22
	$6.9\cdot 10^6$	23		$2.8\cdot 10^7$	23
	$1.2\cdot 10^3$	18		$3.2\cdot 10^3$	19
	$1.2\cdot 10^5$	22		$3.0\cdot 10^5$	22
	$1.2\cdot 10^6$	23		$3.0\cdot 10^6$	23
	$1.2\cdot 10^7$	23		$3.0\cdot 10^7$	25

Future work

- Parallelization (High performance calculus)
- Extension to formulations modelling dielectrics (PMCHWT)
- Industrial applications with the industrial partner.

List of attended classes

- 01DOBRV – Mathematical-physical theory of electromagnetism (8/09/2022 2022, 20)
- 01DPJRV – Lens antennas: Fundamentals and present applications (7/12/2021, 13.33)
01SFVRV – Metamaterials: Theory and multiphysics applications (01/04/2022, 33.33)
- 01UIZRV – Microwave sensing and imaging for innovative applications in health and food industry (22/3/2022, 33.33)
- 01UJDRV – Integral operators and fast solvers: a cross-disciplinary excursus on the best of FFT's companions (13/09/2022, 35)

POLITECNICO DI TORINO

PhD program in
Electrical, Electronics and Communications Engineering